UNIVERSITY OF AGDER

An empirical study of hedging in the Nordic power market

Kristian Nygård and Vegard Vedvik Holm

Supervisor

Dennis Frestad

This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Department of Economics and Business Administration

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Vegard Vedvik Holm Kristian Nygård

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1. Introduction

1.1 Abstract

Due to the deregulation of electricity, the market for the trading of power has increased considerably over the last twenty years. In this period, prices for electricity have proven to be very volatile, so in an attempt to manage risk and handle the volatility of electricity prices, firms are increasing their focus on risk management.

An important part of risk management is the use of derivative markets to hedge a firm's risk exposure. In this thesis, we look at some of the traditional hedging strategies. The first strategy involves determination of whether it is possible to minimise the variance of a portfolio through the theory of optimal hedge ratio. We also study the returns from various future and forward contracts designed to hedge the same obligation.

The market where this study is conducted is the Nordic power market Nord Pool. This market is one of the longest-standing deregulated markets for electricity trading. We use historical data available from Nord Pool to perform the analysis in this thesis.

1.2 Risk Management

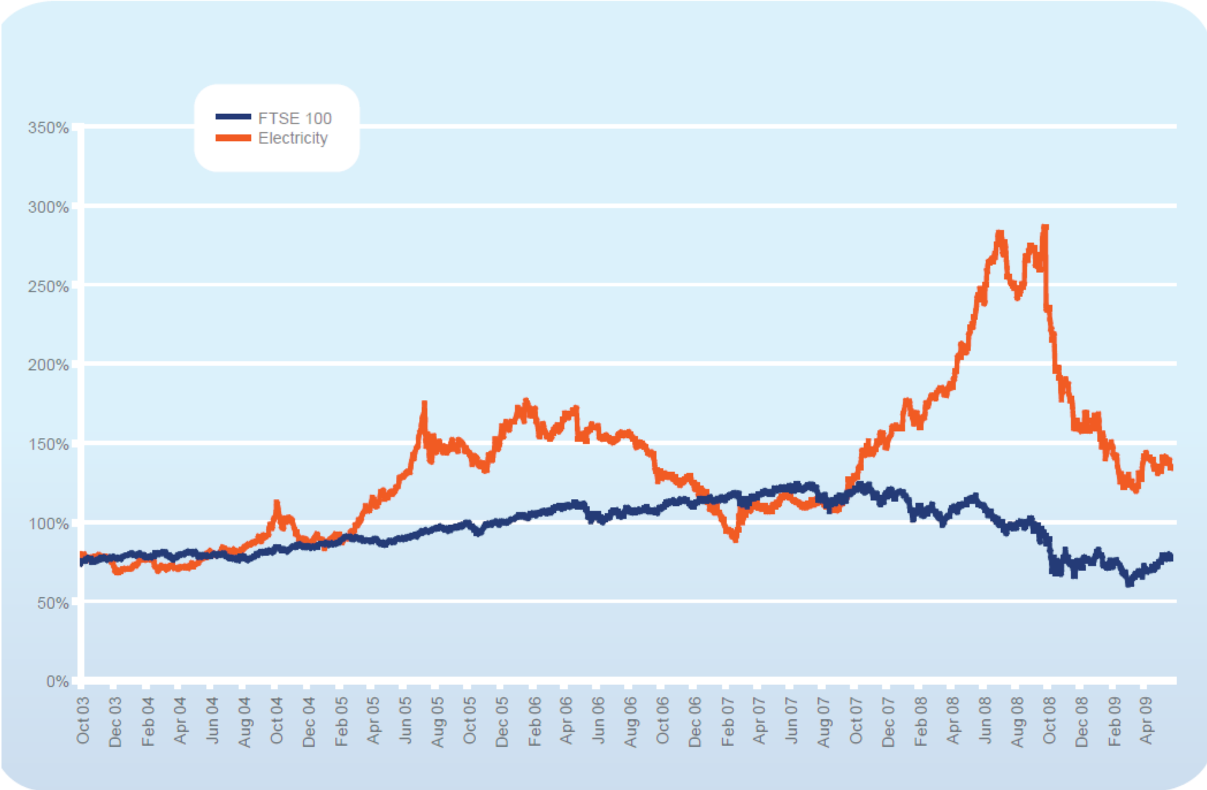
The core of risk management is to assess the uncertainty of the future to make the best possible decision today. We use risk management not to eliminate risk but to understand how different risks may affect a firm. The goal is to minimise losses, to exploit opportunities, and maximise gains given different risks. Risk management is beneficial because it can lead to better decisions, fewer surprises, improved performance and more effectiveness (Veritas 2011). Some firms enlist entire departments that engage risk management as their main task (Stulz 2002).

There are many different ways for a firm to assess and manage different risks. In this thesis, we focus exclusively on the way derivative contracts can be used to manage price risk in the electricity market.

There are two different ways of using derivatives: hedging and speculating. Hedging can be used with derivatives to reduce differing price risks. Power suppliers and power-intensive industries share a common price risk because of fluctuating prices in the power market. To manage this risk, a power-intensive firm can, for instance, use derivatives to hedge its expenses by locking the price at which it purchases power. Another firm in the same market may speculate on the movement of electricity prices and use derivatives to capitalise on its expectations about their movement.

An example of the volatility of electricity prices is shown in graph 1. Volatility represents the risk that the price of a security will change. A higher volatility means that a price is spread out over a larger range. A high volatility also means that the price can change dramatically over a short period of time in either direction. A low volatility represents a steadier price over time. The high volatility of electricity prices is affected by a number of factors, such as the economical climate, the price of other commodities, weather forecasts and natural disasters.

To exemplify the volatility of electricity prices, we show an example for the UK in graph 1. In the UK, electricity price is one of the most volatile indices. From May 2007 to May 2008, the price doubled. In late 2008, the price halved in three months, and historical data shows that it can fluctuate by 2%–5% in a single day.



Graph 1: An overview of the difference in the volatility between the FTSE 100 index and electricity price. FTSE 100 is a share index of the 100 most highly capitalised UK companies listed on the London Stock Exchange (Energy 2009).

We can offer two examples of how volatile electricity prices can affect electricity producers in a negative way.

"During the summer of 1998, wholesale power prices in the Midwest of US surged to a stunning $7000 per megawatt (MWh) from the normal price range of $30–$60 per MWh, causing the defaults of two power marketers in the east coast. In February 2004, persistent high prices in Texas during a 3-day ice storm led to the bankruptcy of a retail energy provider that was exposed to spot market prices." (Deng and Oren)

As the examples show, companies in the power industry are likely to benefit from reducing the risk of their cash-flows through, for instance, hedging their positions to minimise the price risk. The extreme volatility of wholesale power prices implies that even well-capitalised power firms may have power price exposures sufficiently large that adverse price changes could lead to corporate default or bankruptcy. The large capital investments involved in power production and in the distribution of power increase the relevance of risk-related changes in investment incentives.

One of the reasons that electricity is so volatile is that it cannot be stored. The transmission of electricity is limited by physical constraints that are dependent on supply and demand, qualities that fluctuate based on weather conditions as well as prices for other commodities (Energy 2009). In Norway, weather conditions are vital for both the cost of producing power and for the demand for power. The reason is that Norway is very dependent on hydropower, which contributes close to 99% of the total electricity production (Burger, Graeber et al. 2007). Prices in Norway are therefore dependent on rain and snow melting to replenish the water reservoirs. A typical example is that the more rain and snow melts, the lower the electricity price because hydropower suppliers have excess water for the production of electricity. The demand for power is also dependent on the weather when cold winters increase the demand of power (and thus also the price of electricity) substantially.

Norway uses hydroelectricity as its primary source of power. Internationally, it is more common with different power sources such as oil, gas and coal (Energi 2011). The prices of these commodities have an indirect impact on the demand for and supply of electricity. The reason for this is that the increasing prices for these commodities shift the demand for power from these commodities to electricity. In the opposite case, high prices for electricity shift demand away from electricity and towards alternative power sources if this is possible within the market.

Our focus in this thesis is the Nordic power market. Since most of the electricity production in the Nordic power market is a product of hydropower, weather remains the most important factor in the way electricity prices change both supply and demand. However, with increasing globalisation in which power markets are increasingly connected internationally, it is likely that the prices of other power sources will affect the price of electricity in Norway together with the other factors that we have explained. Globalisation also brings with it other factors, such as volatility in financial markets, in interest rates, in exchange rates and in the general economical climate.

Risk management is thus important for all firms in any type of market. The volatility of electricity prices shows that this is especially the case for firms that have earnings that are sensitive to movement in electricity prices. This thesis studies how risk management can be achieved for the Nordic power market through the use of derivatives.

1.3 Tools for risk management

In this chapter, we present and examine different derivatives and explain how they are used.

1.3.1 Derivatives

A derivative contract is a contractual agreement to execute an exchange at some future date (Whaley 2007). The reason that it is called a “derivative” is because the specific agreement derives its value from the price of an underlying asset. Examples of such underlying assets include stocks, bonds, commodities or currencies. For electricity derivatives, the underlying asset is the price of electricity in the market to which the derivative contract refers.

Derivatives are very useful for risk management purposes because they represent low-cost transactions for managing different types of risk (Hillier, Grinblatt et al. 2008). The trading costs are generally lower for derivatives contracts than for trading with the underlying assets. The reason for this is that in trading with derivatives, the only costs that are incurred are those such as bid/ask spreads and broker commissions (Hillier, Grinblatt et al. 2008). In trading the underlying commodity, other costs are incurred, such as transport and storage costs, which are generally much higher than the costs of trading with derivatives. It can even be said for electricity that it is impossible to trade the underlying commodity by storing and transporting it. This issue will be discussed later in the thesis.

There are a number of different types of derivatives, such as forwards, futures, swaps and options. The focus in this thesis will be forward and future contracts. A forward contract represents the obligation to buy or sell a security or commodity at a pre-specified price (known as the forward price) at some future date (Bodie, Kane et al. 2009). Forward contracts are traded at an exchange but can also be traded over-the-counter between two parties. For a forward traded at an exchange, the particular forward price will usually change throughout a trading period as the trade is conducted. The movement of a forward price changes the value of the contract and results in a daily gain or loss for the two parties involved in the contract. However, the daily gains or losses are accumulated and are not paid before the agreed delivery time of the contract. Forward contracts traditionally have longer maturity than future contracts, and it is possible to purchase a right to buy or sell a commodity as much as five years before the actual delivery.

Future contracts are a variant of forward contracts and are also traded within specific futures markets. The main difference between futures and forwards is in how each treats the daily gains and losses from changes in the future price of a contract. Instead of accumulating the difference, the daily gains and losses are debited and credited from the seller and purchaser of a contract for each day (Bodie, Kane et al. 2009). Future contracts have a shorter maturity than forward contracts, so it is possible to trade futures as early as one day before the delivery for some commodities.

A trader who purchases a future or forward contract takes a long position on a commodity. By doing so, he/she is committed to purchase the commodity at the delivery date. A trader who sells a future or forward contract takes a short position on a commodity. He/she is committed to deliver that commodity on the delivery date (Bodie, Kane et al. 2009). In this thesis, we will use the terms long and short positions to indicate the different hedge positions.

Common to both futures and forwards is the fact that, at maturity, the price paid is the spot price for the underlying asset. It is possible to look at a forward or future price as an indication of what a trader predicts the spot price will be in the future. Another interpretation is that the forward/future price is a price at which a firm is satisfied to sell or purchase a commodity. The firm can use the market as a risk management tool to lock that price today to avoid uncertainty in the spot price of a commodity. At the delivery time specified by the contract, the long or short position will either gain or lose money by holding the contract given that the contract is not perfectly priced compared to the spot price. The gain or loss is indicated by the difference between the spot price and the forward/future price. If the spot price is higher than the forward/future price at delivery, the long position receives the monetary difference that the short position must pay. The roles change if the situation is reversed.

1.3.2 The pricing of a forward contract with traditional commodities

There is a fundamental idea behind the use of all traditional derivative models. Traditional commodities consist of, for instance, grain, wheat, oil and others. The idea driving derivatives is that it is always possible to develop a portfolio consisting of the underlying asset and a risk-free asset that tracks the future cash-flow of the derivative (Hillier, Grinblatt et al. 2008). All valuation models have a perfect market assumption so that there are no arbitrage opportunities. This assumption means that a derivative must have the same value as the tracking portfolio. The general concept of valuation models is that it is irrelevant whether you hold a cash position or derivative position because at maturity, the value of each is the same.

The traditional method of valuing derivatives for underlying commodities is based on this general idea but with the addition of the concept of cost of carry. Cost of carry tries to catch all the costs which are related to the practice of holding a commodity over a lengthy period (Hillier, Grinblatt et al. 2008). Such costs include, for instance, storage and obsolescence costs. For some commodities, there is a negative cost of carry that is referred to as a convenience yield. A convenience yield may occur if the costs of acquiring a commodity are greater than the costs of holding it, such that holding a commodity becomes beneficial for its owner.

Equation 1 shows how to price a forward contract for an underlying commodity with the traditional theory. Given a no arbitrage assumptions, the contract is a zero present value investment.

$$F_0 = S_0\left(1 + r_f\right)t + C \tag{1}$$

F_0: The forward price at time 0.
S_0: The spot price of a commodity at time 0, also called cash price.
R_f: A risk-free rate of a security.
C: The cost of carry for the commodity. If negative, we use the term convenience yield.
t: The time left before the forward matures.

1.3.3 Hedging a traditional commodity with a forward contract

The traditional theory assumes perfect market conditions at all times so that it is irrelevant whether an investor purchases a forward contract or instead simply buys the underlying commodity right away. This theory does not always hold, and this is the reason that firms use derivatives to hedge their positions.

To illustrate how traditional commodities are hedged, we can use an example of a firm supplying oil at a fixed rate that needs to purchase crude oil to be able to fulfil its obligation. To hedge the price of the crude oil, the firm purchases forward contracts.

The point of this is to lock in the price at which crude oil was purchased to eliminate the price risk that fluctuating spot prices of crude oil may have. As long as the locked forward price is lower than the fixed output price, the firm is guaranteed a profit when they are committed to deliver.

1.3.4 Hedging a traditional commodity with a future contract

Equation 1 may also be applied to future contracts. The differences between the two contract types have already been discussed earlier in this chapter. We can change the previous example and presuppose that the oil company already has the oil it needs but wishes to sell this oil in one year. To secure its position, the company sells futures for the total output. Because of the mark-to-market settlement, the hedge ratio must be less than 1. The reason for this is that one must account for the interest earned on the mark-to-market settlements (Hillier, Grinblatt et al. 2008). This can be explained from equation 1. Consider that the spot price of oil is €1 and the risk-free rate is 5%. The future price from equation 1 is computed to be €1.05. In this example, the cost of carry is ignored. If the spot price increases to €1.10, we achieve a gain from holding the oil of €1 – €1.10 = €0.1. However, we lose €0.155 because we have sold a future with a lower spot price. The solution to this problem is called tailing the hedge, a situation in which the hedge ratio chosen is calculated by the number of financial contracts divided by the risk-free interest rate for that year. By using this method, one keeps the net value of the future position and the commodity cash position at zero throughout the hedging period (Figlewski, Landskroner et al. 1991).

1.3.5 Applying traditional derivative pricing theory to electricity derivatives

The fundamental problem with electricity, as opposed to other commodities such as crude oil, grain and others, is that it cannot be stored. Some may argue that this is not true for hydroelectricity because it may be stored in the form of water until a producer wishes to generate power. However, hydropower in itself is not electricity, a fact that makes it impossible to apply a relationship between this form of the commodity and electricity spot prices.

The traditional pricing formula shown in equation 1 requires that it be possible to store the commodity and through this, to link the forward price with the spot price under a no-arbitrage condition (Bessembinder 2002). The same article states that arbitrage strategies, such as holding an underlying asset at spot price and storing it for a sale at forward price, will not be possible with electricity because it cannot be stored.

Electricity derivative contracts do not have a specific delivery date but instead involve a delivery period, a feature that is not accounted for in the traditional pricing formula.

The fact that electricity cannot be stored has consequences for convenience yield and for tailing the hedge. For convenience yield, it is impossible to quantify any type of storage costs or benefits from holding the electricity (Shawky, Marathe et al. 2003), and this calculation therefore cannot be used in connection with electricity. This difficulty also means that tailing the hedge is not possible.

1.3.6 Pricing models for electricity derivatives

There is a good deal of available research on models that attempt to capture the common features of electricity and through this, attempt to create models for pricing the forward and future contracts correctly.

One feature is the notion of the long run mean reverting process of electricity prices. The important element here is that in the long run, the cost of production dictates how the price moves (Cartea 2005) so that over time, the spot prices will produce a long term mean. The most notable contribution to this process is by Eduardo Schwartz (1997), who used a stochastic model that included the Ornstein-Uhlenbeck process that sought to catch the mean reversion in spot prices. His model has later been extended to include the seasonal component of spot price movements (Lucia 2002). The data analysis in this paper involved a test at Nord Pool that showed that spot prices are relatively higher in the winter than the summer. Lucia et al showed that the spot prices are increasingly volatile during the summer when compared with the winter.

The last point is often referred to as “spikes” in the spot price. These spikes refer to sudden unpredictable movement in spot prices for a single day where prices skyrocket before normalising themselves within the next few days (Deng 2000). In the same article, Deng et al refers to an example of spikes from electricity prices in the USA. Prices soared from $50 MWh to $7000 MWh in one day before returning to the initial value within a few days. The reasons for these spikes include, for instance, sudden demand that exceeds supply due to weather changes and forecasting error by electricity producers (Lu 2005). Shijie Deng’s stochastic models are the best known formula for the prediction of these spikes by illustrating mean-reversion-jump-diffusion models that attempt to capture both the mean reversion and sudden jumps in the spot price (Deng 2000).

Our focus in this thesis is not to present or test the discussed models individually, but to show the way that they provide insight into the common features underlying spot price movements. This has an impact on the pricing of electricity derivative contracts and thus also on the cash-flows of derivative contracts.

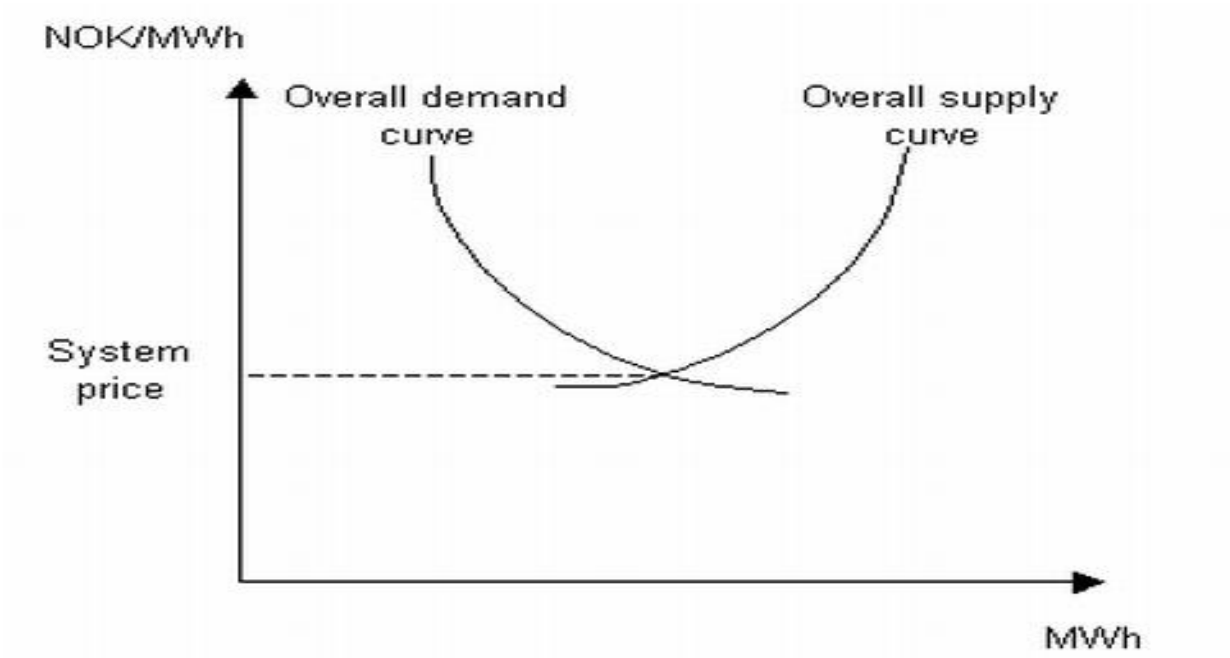
1.5 Nord Pool – Background and Purpose

The background for the Nordic power market and for Nord Pool is determined in part by the results of the LAW OF ENERGY ACT in 1990, in which the legal foundations were laid to deregulate the Norwegian power market (OED 1991). The act was motivated as the result of dissatisfaction with how the regulated market worked. The power price was set centrally, which caused inefficient production because there were no incentives for power producers to be cost efficient (Bye and Hope 2005). Bye et al states there were inefficiencies in the market due to differing prices for different regions. Regions with oversupply created lower prices for local customers by charging higher prices in regions with undersupply. To rectify these problems, a spot marked was established in 1991 in which anyone could purchase power. The marketplace was originally called Statnett and remained so until Sweden deregulated their power market in 1996. Together with Norway, Sweden established a common marketplace for the trade of power with the name Nord Pool. Finland (1998) and Denmark (2002) have since joined Nord Pool, creating the Nordic power market we know today. The purpose of Nord Pool is to encourage the most efficient use of energy possible.

1.5.1 Nord Pool – Market overview and product specification

Since its establishment, Nord Pool has grown to become the largest market for electric energy in the world. In 2009, the organisation had a turnover of 288 terawatt (TWh), representing a value of EUR 10.8 billion, which was 70% of the total consumption of electricity in the Nordic countries (Nordpoolspot 2011). Nord Pool today consists of two different entities: Nord Pool Spot AS, which runs the physical delivery spot markets; NASDAQ OMX Commodities Europe, which oversees the financial market and which runs all clearing services.

Nord Pool Spot AS (Elspot) is the centre for all physical delivery power contracts. Market participants use the market to buy and sell power on a daily basis. The system of trade is strictly regulated and takes the form of an auction. All market participants send their bid/ask prices for each hour of the following day by 12:00 of the day before. By a “bid”, we here refer to a specific offering price that a purchaser is willing to pay for power at a specific hour. The “ask” is the price at which a seller is willing to sell power at a specific hour. On the basis of the bid/ask received, the system operator at Elspot derives a supply and demand curve that serves as an economic model to determine the system price for each hour of the following day. The system price is denominated as price per MWh, which is the price the market participants pay or receive for the power the following day for each hour. This is illustrated in graph 2.



Graph 2: This graph, from the trading manual of Nord Pool (Nord Pool 2010), shows how the Nord Pool system price is calculated as the equilibrium between supply and demand for each hour daily.

The system price represents the market equilibrium and also works as the underlying asset for the financial contracts at Nord Pool ASA. Nord Pool is split up into different regional areas: Norway consists of five, Denmark two, while Sweden and Finland have only has one each (Nordpoolspot 2011). The system price is the reference for all of these regional areas of Nord Pool, given that the capacity grid is not exceeded. A capacity grid means the overall capacity of supplying power in one specific regional area. If the system operators in a specific regional area find that some of the areas have a surplus or a deficit in capacity due to bottlenecks, those areas will be assigned an independent area price which differs from the system price. The reason for this is to ensure that each regional area remains technically stable. In this thesis, we will not focus on area prices but will instead use the average system price. The average system price can also be written as the average spot price since they both mean the same.

NASDAQ OMX Commodities Europe offers a wide range of different financial derivative products for trade and risk purposes. There is no physical delivery of power; all positions are closed out by cash settlements. Energy derivatives can be purchased with two different delivery times. Base contracts are delivered from 00:00–24:00, while Peak contracts are delivered from 08:00–20:00. This has implications for the reference of the underlying asset because the spot price reference for a peak-load will only refer to the system price between 08:00–20:00, while base-load contracts refer to prices at all hours.

1.6 Trading at Nord Pool with futures and forwards

In this section, we present the different future/forward contracts that are available for trade at Nord Pool. We also explain how the cash-flow from using these contracts is calculated for both mark-to-market settlement and spot price reference.

1.6.1 Futures

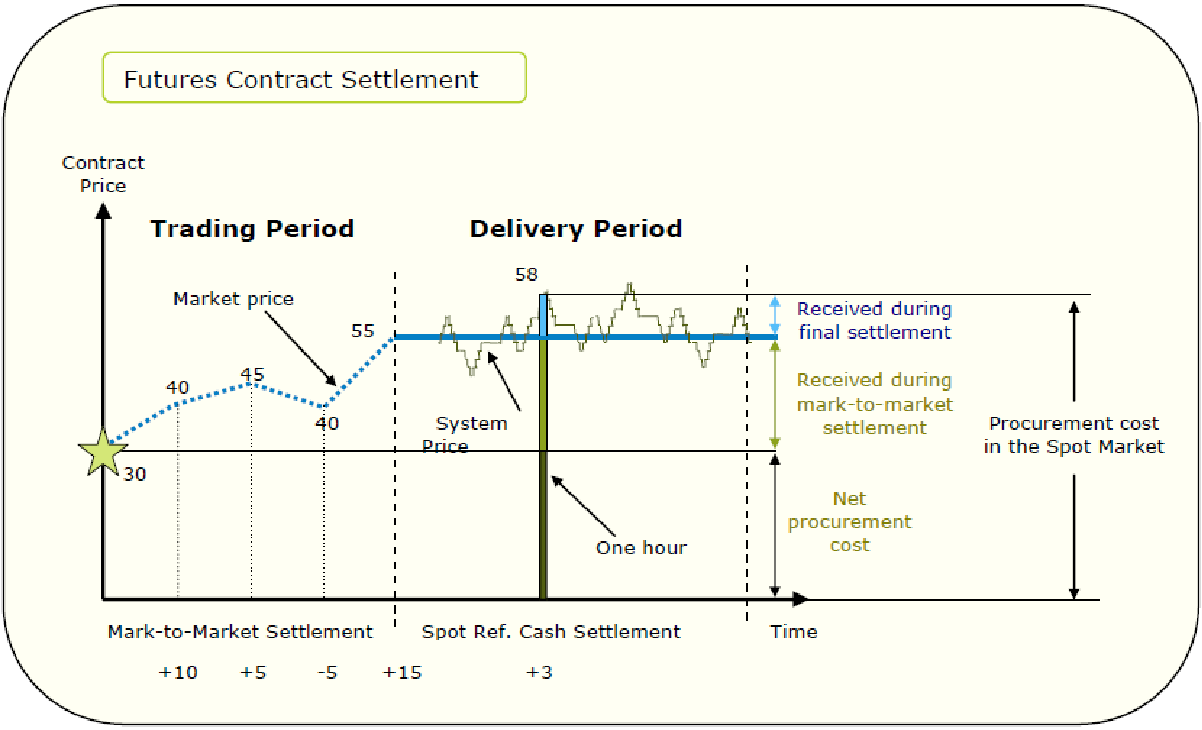
TYPE OF AVAILABLE FUTURE PRODUCTS				
Load	No. of days	Hours	No. of weeks	Hours
Base load	2 to 9	24	6	168
Peak load	N/A	N/A	5	60

Table 1: An overview of the available future products at Nord Pool for both base load and peak load. No. of days/weeks refers to the length of time before the delivery period of a contract can be traded. Hours refers to the number hours of which a contract is composed. For a base-load contract the total hours represent 24 hours for each day to which a contract refers while peak-load contracts represents 12 hours for each day. A weekly base-load contract is therefore the product of 24 * 7 = 168. Since one hour equals one MWh, the number of hours also illustrates how many MWh there are in each contract.

Table 1 shows what types of futures products are available for trade. The daily futures have a nine-day trading period, while the weekly futures have a six-week trading period for base load contracts before the delivery period. In this thesis the focus is on base load contracts, so we will limit any discussion of the peak load contracts to a presentation of the different types available.

During the trading period, daily mark-to-market settlements are realised by the market participants. When the trading period expires, there is a delivery period during the subsequent day or week (depending on the contract purchased). In the delivery period, there are hourly spot reference cash settlements between the closing value of the future contract and the Nord Pool system price for each hour. This means that if a base-load daily future is purchased, there are 24 such references; for a base-load weekly future contract there are 168 references.

Graph 3 shows how such a trade works from the trading guide of Nord Pool (Nord Pool 2010). A future is bought for €30 and during the trading period, the price of the future contract increases to €55, which is also the closing value of the future. Daily mark-to-market settlements are received throughout the trading period, so that the holder of the future receives a gain of €55 - €30 = €25. In the subsequent delivery period, there are hourly spot price references. The spot price reference uses the Nordic average system price for each hour as the underlying asset. In this example, one specific hour is used to illustrate how this is done. In that hour, the system price at Nord Pool is €58 so that the holder receives €58 - €55 = €3 in spot reference cash settlement. This spot price reference is repeated for all of the hours in the delivery period of the contract.



Graph 3: An example from the Nord Pool trading guide (Nord Pool 2010) that shows how daily settlements and spot price references are conducted. In this example, it is assumed that a market participant has bought a future contract.

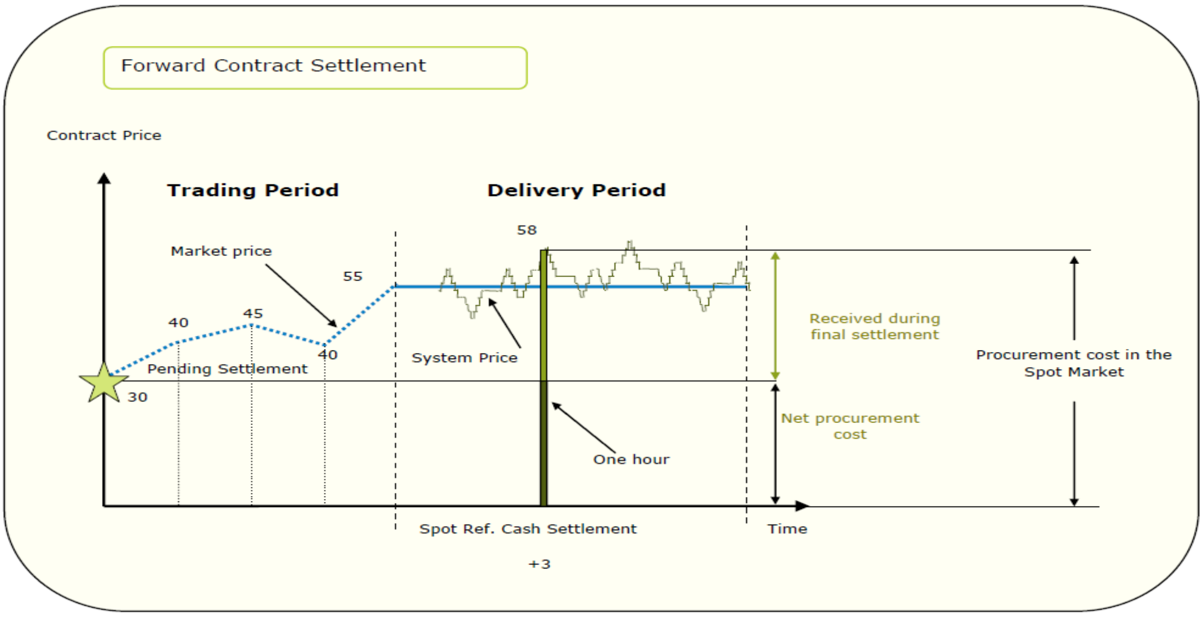
1.6.2 Forwards

TYPE OF AVAILABLE FORWARD PRODUCTS						
Load	No. of months	Hours	No. of quarters	Hours	No. of years	Hours
Base	6	672–744	8 to 11	2159–2209	5	8760–8784
Peak	2	240–276	3	768–792	1	3132–3144

Table 2: An overview of all available forward products at Nord Pool. Because each month and year is different with regard to the number of days included, there are differences in the overall hours in each contract. The row of hours includes a bracket [X – Y] that indicates that a contract has a minimum of X hours and a maximum of Y hours. Aside from this difference, we use the same indicators as with our treatment of future contracts.

Table 2 shows all the different forward contracts that are available for trade at Nord Pool. The procedure for a forward contract is very similar to that of a future and will therefore not be repeated here. However, the distinct difference is that a daily mark-to-market settlement is not received as a result of holding a forward during the trading period. The change in value is instead accumulated until the delivery period, where for each hour the daily mark-to-market settlement is done. In the delivery period, hourly system price references are calculated as described previously for futures.

Graph 4 graphically uses the same example from the Nord Pool trading guide (Nord Pool 2010), where now a forward is purchased instead of a future. The accumulated value change is referred to as “pending settlement”, which is not received until the delivery period when the holder receives €55 - €30 = €25 for each hour. The procedure for calculating spot-price-reference-per-hour is the same as with the futures example described above.



Graph 4: An example from the Nord Pool trading guide (Nord Pool 2010) in which a forward is purchased. It is almost identical to the future contract, but there are no daily settlements in the trading period.

2. Theory

The theory chapter consists of the presentation of two different hedge strategies. We first present the theory of the risk-minimising strategy that makes use of an optimal hedge ratio. Following this, we present a theory for the calculation of the cash-flow of a hedge strategy with delivery.

2.1 Optimal hedge ratio

In the following section, we demonstrate a model that calculates an optimal hedge ratio different from the hedge ratio in traditional hedge theory. The optimal hedge ratio is used as a tool to minimise the variance of a specific position. Later in the thesis, we study the optimal hedge ratio in the futures market in the Nordic power market and determine whether it minimises the risk of the closing value of a hedge. In this study, we disregard interest payments and other costs that incur through hedging that may affect the value of the hedge.

An article that takes a different approach to the futures market through the use of portfolio theory is "The Hedging Performance of the New Future Markets" by Louis Ederington (1979).

Ederington defines a hedge as one in which the seller or buyer of the future contract cancels his delivery commitment by buying or selling a contract for the same futures prior to delivery (Ederington 1979). The main point of his article is to eliminate the notion that to have a perfect hedge it is necessary to fully hedge a commodity and to propose instead that it might be optimal to keep a lower hedge ratio. The hedge ratio shows the relationship between how much we have invested in the future market and in the spot market. With an optimal hedge ratio, we find the perfect match between the future and spot markets that minimises the variance; by minimising the variance, we reduce the risk on our position.

We will first show how Ederington calculates the optimal b before showing how the optimal b can be used when calculating the value of a hedge.

The gain or loss of an unhedged position is:

$$X[P_s^2 - P_s^1]$$

The gain or loss of a hedged position is:

$$X\{[P_s^2 - P_s^1] - [P_f^2 - P_f^1]\}$$

X, the number of units
$\boldsymbol{P_s^1}$**, spot price at time 1.**
$\boldsymbol{P_s^2}$**, spot price at time 2.**
$\boldsymbol{P_f^1}$**, future price at time 1.**
$\boldsymbol{P_f^2}$**, future price at time 2.**

The perfect hedge changes when the basis is equal to zero, where the basis is the difference between the future and spot prices:

$$\{(P_f^2 - P_s^2) - (P_f^1 - P_s^1)\}.$$

The basis shows the potential gains or losses in the hedge strategy, and it also adds some risk to the position. The basis risk is the mismatch between the spot and futures position.

U represents the value of the unhedged position, while R is the value of the hedged position:

$$Var(U) = X_s^2\sigma_s^2 \tag{2}$$

$$E(U) = X_s E[P_s^2 - P_s^1] \tag{3}$$

$$Var(R) = X_s^2\sigma_s^2 + X_f^2\sigma_f^2 + 2X_s X_f \sigma_{sf} \tag{4}$$

$$E(R) = X_s E[P_s^2 - P_s^1] + X_f E[P_f^2 - P_f^1] \tag{5}$$

$\boldsymbol{X_s}$**, quantity held in the spot market.**
$\boldsymbol{X_f}$**, quantity held in the future market.**
$\boldsymbol{\sigma_s^2}$**, variance of the possible change in the spot price from** $\boldsymbol{t_1}$**to** $\boldsymbol{t_2}$**.**
$\boldsymbol{\sigma_f^2}$**, variance if the possible change in the future price from** $\boldsymbol{t_1}$**to** $\boldsymbol{t_2}$**.**
$\boldsymbol{\sigma_{sf}}$**, the covariance between the spot and future price.**

Traditional theory prior to Ederington stated that the hedge ratio was always equal to 1, so that

$$X_f = -X_s.$$

With the hedged position, there is a possibility that the hedged portfolio may be either completely or partly hedged. To illustrate this, Ederington uses the formula $b = \frac{-X_f}{X_s}$, which shows the proportion of the spot position that is hedged. X_s and X_f have opposite signs, meaning that b is usually positive.

$$Var(R) = X_s^2\{\sigma_s^2 + b^2\sigma_f^2 - 2b\sigma_{sf}\} \qquad (6)$$

$$E(R) = X_s\{E(P_s^2 - P_s^1) - bE(P_f^2 - P_f^1)\}$$

The next step is to show the way the expected spot price change is affected by the ratio of the hedge. To do this, we insert this equation $bE(P_s^2 - P_s^1) - bE(P_s^2 - P_s^1)$ into E(R), and then get

$$E(R) = X_s\{(1-b)E(P_s^2 - P_s^1) + bE(P_s^2 - P_s^1) - bE(P_f^2 - P_f^1)\} \qquad (7)$$

We include $E(S) = E(P_s^2 - P_s^1)$, which represents the expected change in the spot price. $bE(P_s^2 - P_s^1) - bE(P_f^2 - P_f^1)$ we set as $bE(\Delta b)$, where E(Δb) is the change in the basis, and we get

$$E(R) = X_s[(1-b)E(S) - bE(\Delta b)] \qquad (8)$$

If the expected change in the basis is zero, then clearly the expected gain or loss is reduced as b→1.

We will now examine, when holding X_s constant, what happens to the variance and expected return when b changes. To do this, we derive Var(R) and E(R) with respect to b,

$$\frac{\partial\, Var(R)}{\partial b} = X_s^2\{2b\sigma_f^2 - 2\sigma_{sf}\}$$

$$\downarrow$$

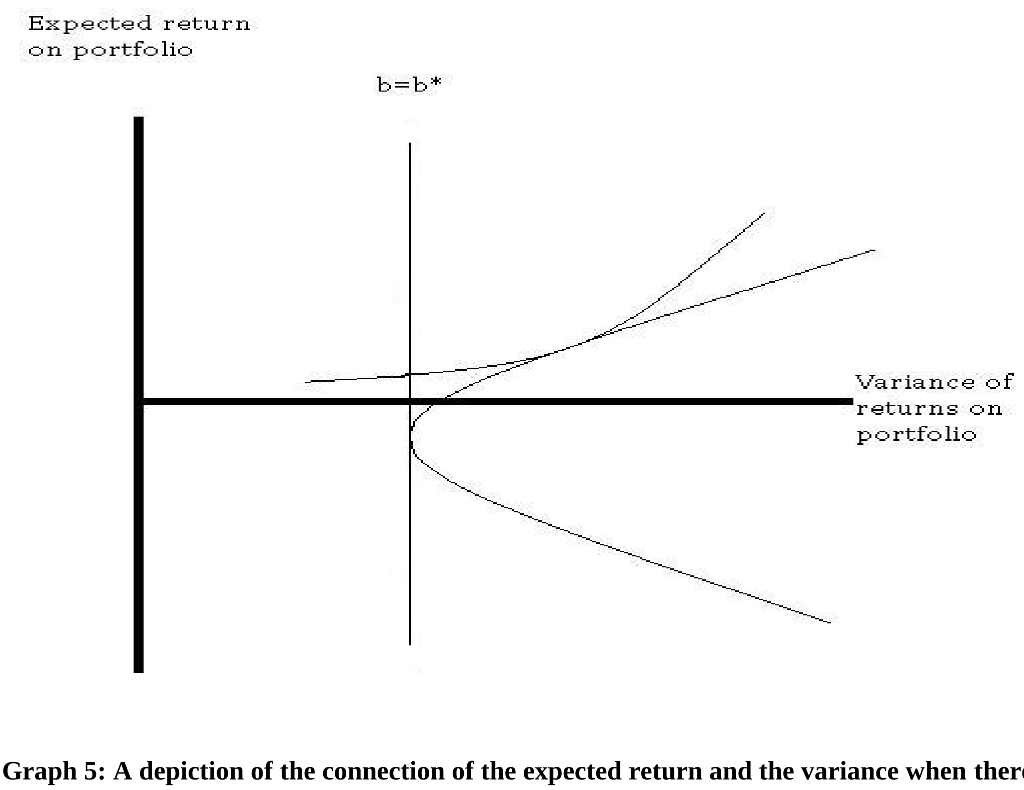
$$b^* = \frac{\sigma_{sf}}{\sigma_f^2} \tag{9}$$

This means that, using b* instead of b, we minimise the variance of the position.

$$\frac{\partial E(R)}{\partial b} = -X_s[E(\Delta b) + E(S)] \tag{10}$$

The sign of equation 10 determines whether expected return moves clockwise or counterclockwise around the locus when b increases. This is illustrated in graph 5.

The optimal b* does not need to be equal to one as it does in traditional hedging theory. If b* is greater than one, investors should take a larger position in the futures, and if it is below one, investors should reduce investment in futures.



Graph 5: A depiction of the connection of the expected return and the variance when there is an optimal b (b*). When the variance is minimised by the optimal hedge ratio, the expected return decreases. If we want a higher expected return, the variance goes up, meaning that we take more risk into our position (Ederington 1979).

2.2 Hedge effectiveness

We will now examine the effectiveness of hedging. Hedging effectiveness shows how much risk can be eliminated from a position by hedging with an optimal hedge ratio. Using the variance, we calculate the effectiveness between two positions with different hedge ratios. To illustrate the effectiveness of this technique, we use an unhedged and an optimal hedged position.

$$e = 1 - \frac{Var(R*)}{Var(U)} \tag{11}$$

E, hedging effectiveness.
Var(U), variance of an unhedged position.
Var(R*), variance of a minimum risk portfolio with b=b*.

We can make use of equation 6, which shows us the variance of a hedged position. Now we substitute equation 9 into this, and we get

$$Var(R*) = X_s^2\{\sigma_s^2 + \frac{\sigma_{sf}^2}{\sigma_f^2} - 2\frac{\sigma_{sf}^2}{\sigma_f^2}\}$$

$$= X_s^2(\sigma_s^2 - \frac{\sigma_{sf}^2}{\sigma_f^2}) \tag{12}$$

Putting [12] and [2] into [11], we get the new formula for hedging effectiveness (e),

$$e = \frac{\sigma_{sf}^2}{\sigma_s^2\sigma_f^2} \tag{13}$$

The hedging effectiveness shows how much risk can be eliminated from a position by hedging with an optimal hedge ratio compared to a similar portfolio with a different hedge ratio. It is measured in percentage.

2.3 Value of a hedge:

Ederington uses his model to evaluate futures markets for corn, wheat and treasury bills and shows that the variance of a portfolio can be minimised with an optimal hedge ratio different than 1. The same model for finding an optimal b has been used by Tanlapco, Lawarrée and Liu (2002) to examine different hedge strategies in the American electricity market. Using historical data, Tanlapco et al examine the possibilities of reducing the variance in electricity trading when using different futures contracts.

Using the optimal hedge ratio, these authors study the chance of minimising the variance of the value of the hedge and thus of reducing the variance of either purchasing or selling electricity in the market.

When calculating the value, they use the formula:

$$Value\ of\ the\ hedge = S_{t+1} + b(F_t - F_{t+1}) \tag{14}$$

S_t, spot price at t+1 that we will sell 1MWh of electricity.
B, optimal hedge ratio calculated with the theory of Ederington.
$F_t - F_{t+1}$, future price at t and t+1.

The equation shows the value for each MWh to be sold at time t+1.

Later in the text, we use the model of optimal hedge ratio and value of the hedge and determine how this model works for the Nordic electricity market. One contract at Nord Pool contains 168 MWh, but in our study of the optimal hedge ratio, the contract represents 1 MWh to simplify the case and the results. We determine whether there is a possibility of obtaining an optimal b that minimises the value of the hedge and how this will change the hedging effectiveness of the position.

2.4 Hedging with delivery

The presentation of the risk-minimising hedge strategy involves optimising the hedge ratio and selling the derivative before the commodity delivery period. In this section, we present a framework to study the returns of a position from a fully-hedged obligation. We do this by acquiring a future or forward at different times during a contract's trading period and by keeping it until the contract matures. Keeping the contract until it matures means that we hold the contract all the way through the delivery period. We make use of the same assumptions as previously for this hedge strategy. This means that we continue to disregard the effect interest payments have on the daily mark-to-market settlement for future contracts. We also disregard the broker costs that occur from trading at Nord Pool.

In our context, "returns" means receiving the most from selling power, and paying the least for purchasing power. The market participant uses only the Nord Pool spot market to sell or purchase power but uses the corresponding derivative market to hedge his/her obligations. The focus in this thesis is a supplier that produces and sells power at the current market price of Nord Pool each day but uses the corresponding futures market to hedge delivery. However, the framework can also be utilised for testing a long position.

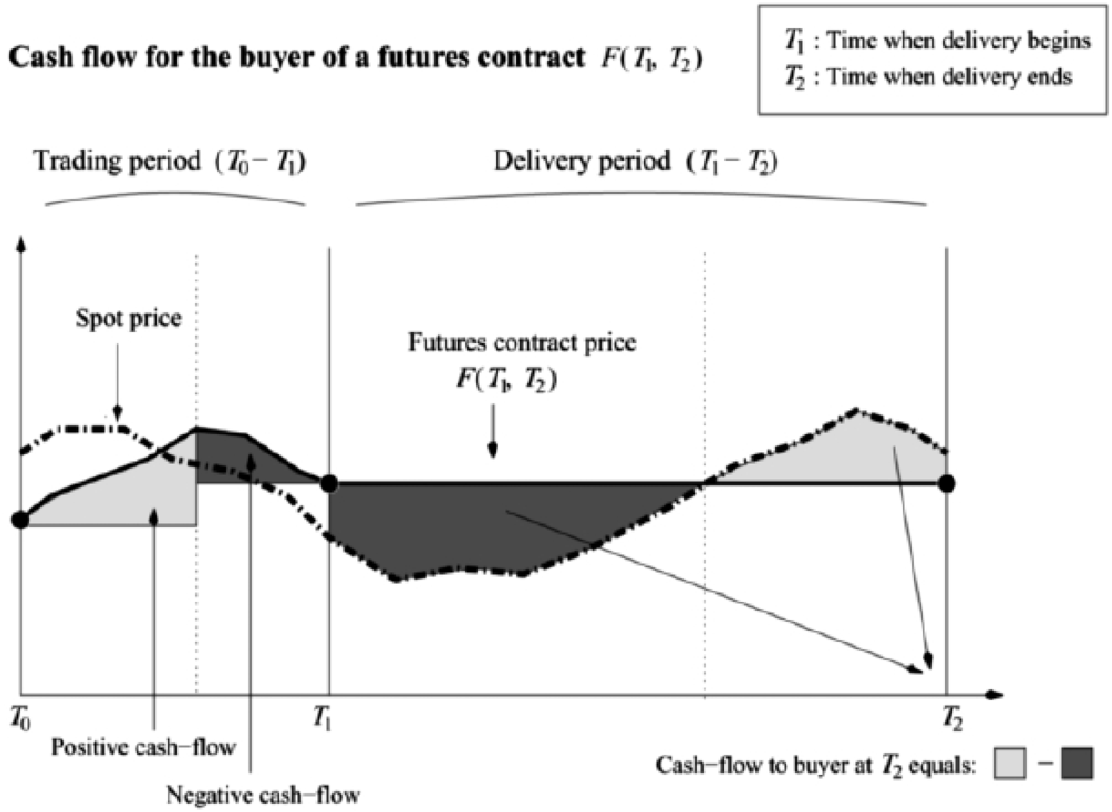
The key aim is to show a framework that makes it possible to test whether hedging at Nord Pool with differently timed hedge strategies yields better results when compared with not hedging at all. This study is performed ex-post, with historical data from Nord Pool.

The typical market participants that seek to use hedging are electricity producers selling contracts and energy intensive industries purchasing contracts (Fleten 2003). Forward and future prices at Nord Pool are a result of changes in supply and demand for the underlying asset (electricity) throughout the trading period of the contract (Fleten 2003). The prices will primarily be a result of what the firms and speculators predict the average spot price will be at the delivery time of a contract. These predictions will be the product of a sum of factors, such as weather and power demand, as we mentioned in the chapter 1.2. The closing price of a contract can therefore be thought of as a predictor of what the market participants think that the average system price will be at the delivery period of a specific contract.

The problem for a hedger or speculator when wishing to maximize returns consists of determining what the price of a contract is when it is acquired when compared with the average spot price in the delivery period. This problem may be simplified into two smaller parts. The first is determining the change in the contract price from the time of acquisition to the final closing price prior to the delivery period. The second part consists of determining how well the closing value of a contract corresponds to the average spot price in the delivery period.

In the following, we will present the way cash-flow from a hedge can be calculated for a short and long position for both futures and forwards. We will also show how correlation can be used as a tool to study the strength between futures and spot prices.

2.5 Hedging with delivery with future contracts



Graph 6: An overview over a cash-flow development of a future contract at Nord Pool from acquisition and until expiration (Fleten 2003).

Graph 6 illustrates an example of how a cash-flow of a hedged position with futures develops from the trading period until the contract matures. The price of the contract at a specific time during the trading period is a result of trading between the market participants.

$$Future\ price = F(t_x, T_1, T_2) \tag{15}$$

t_x, an arbitrary trading day between T_0 and T_1 where $t_0$is the first trading day. The number days that different types of futures are tradable is shown in the chapter 1.6.
T_1, The first day on which the delivery begins. For simplicity, we also use F(T_1) as the closing price of a contract on the final trading day.
T_2, The last day of the delivery.

With arbitrary values of t_x, equation 15 can be used to show the way the future price changes during the trading period. The reason that T_1 and T_2 are included is to show that the future price applies for that specific delivery period.

Because we can either hold a short position or a long position, the future and spot prices must have different signs. This is required if the framework is to be used for both long and short positions with the same equations.

For a short position we use:

$$-F_x = S_x$$

For a long position we use:

$$F_x = -S_x$$

F_x, The future/forward price
S_x, The spot price.

Changes in the future price affect the cash position of the short and long positions of a contract through daily mark-to-market settlements. This is shown in graph 6, where during the trading period, an increase in the future price yields a positive cash-flow for the long position, while a decrease in the future price yields a negative cash-flow. The short position in the same contract will have an exactly opposite cash-flow.

$$Daily\ mark\text{–}\ to\text{–}\ market\ settlements = \Big(-(F\big(t_x^*, T_{1,}T_2\big)) + \ \big(F(t_x, T_1, T_2)\big)\Big) *\ h_t \tag{16}$$

t_x^*, the specific trading day on which a future is purchased or sold and the corresponding price of the future contract at that time (F). The daily mark-to-market settlement is repeated for each new trading day until the contract expires. Note that for each new trading day after taking a position in a contract, the last paragraph ($F(t_x, T_1, T_2)$) will be used as the new basis for settlement.
h_t, the total number of hours of electricity specified in the contract.

To calculate the total mark-to-market settlement of a contract in the trading period, we calculate the difference in the contract price when acquired compared with the closing price of a contract on the last trading day.

$$\begin{aligned} &Total\ mark\text{–}\ to\text{–}\ market\ settlement \\ &\quad = \left(-(F\left(t_x^*, T_{1,}T_2\right)) + (F(T_1))\right) * h_t \end{aligned} \qquad (17)$$

The closing value of the future contract is important because it is used in the spot price reference throughout the delivery period. With the risk minimisation strategy, we avoid this problem by closing out the position prior to delivery. As in the example in graph 6, the long position initially loses money from the spot reference because the spot price is lower than the closing price of the future contract in the delivery period. The reason for this is that the long position investor must pay more for the power because he/she has locked down a higher price in the future contract. If the long position investor had not purchased the contract and through this remained unhedged, he/she could have purchased directly from the spot market at the spot price and avoided the extra costs from the spot price reference. This trend changes toward the latter part of the delivery period, when the spot price is higher than the closing price of the contract. This yields a positive cash-flow for the long position when compared with not hedging. The investor, who has sold this specific contract and because of that holds a short position in the contract, will have cash-flow exactly opposite to that of the long position.

$$Spot\ price\ reference\ per\ day = \left(-\left(F(T_1) - S_y\right)\right) * h_d \qquad (18)$$

S_y, The average spot price of a specific day during the delivery period. Y represents the different delivery days in the delivery period and will be a day between T_2 and T_1. S_0 will, for instance, refer to the average spot price of the first day in the delivery period.
h_d, The number of hours specified for a contract per day. For a base-load contract, this will equal 24 hours, while for a peak-load contract this will equal 12 hours.

The spot price reference is repeated for each day during the delivery period. For a daily future, this means only one day, while for a weekly future, this totals up to seven days.

The total spot price reference is the product of all the spot price references during the delivery period.

$$Total\ spot\ price\ reference = \sum_{y=1}^{n} \left(-\left(F(T_1) - S_y\right)\right) * h_d \qquad (19)$$

N, refers to the number of delivery days a contract has specified and refers to all delivery days from T_1 to T_2.

The derivative markets at Nord Pool do not include physical delivery of power since all Nord Pool contracts are settled financially. This means that, within our context, the short position sells power at the spot marked for the average daily spot price in the delivery period. Likewise, the long position will buy at the given average spot price in the market.

$$Cash - flow\ per\ delivery\ day\ fulfilling\ obligation = S_y * h_d \qquad (20)$$

$$Total\ cash - flow\ by\ fulfilling\ obligation = \sum_{y=1}^{n} S_y h_d \qquad (21)$$

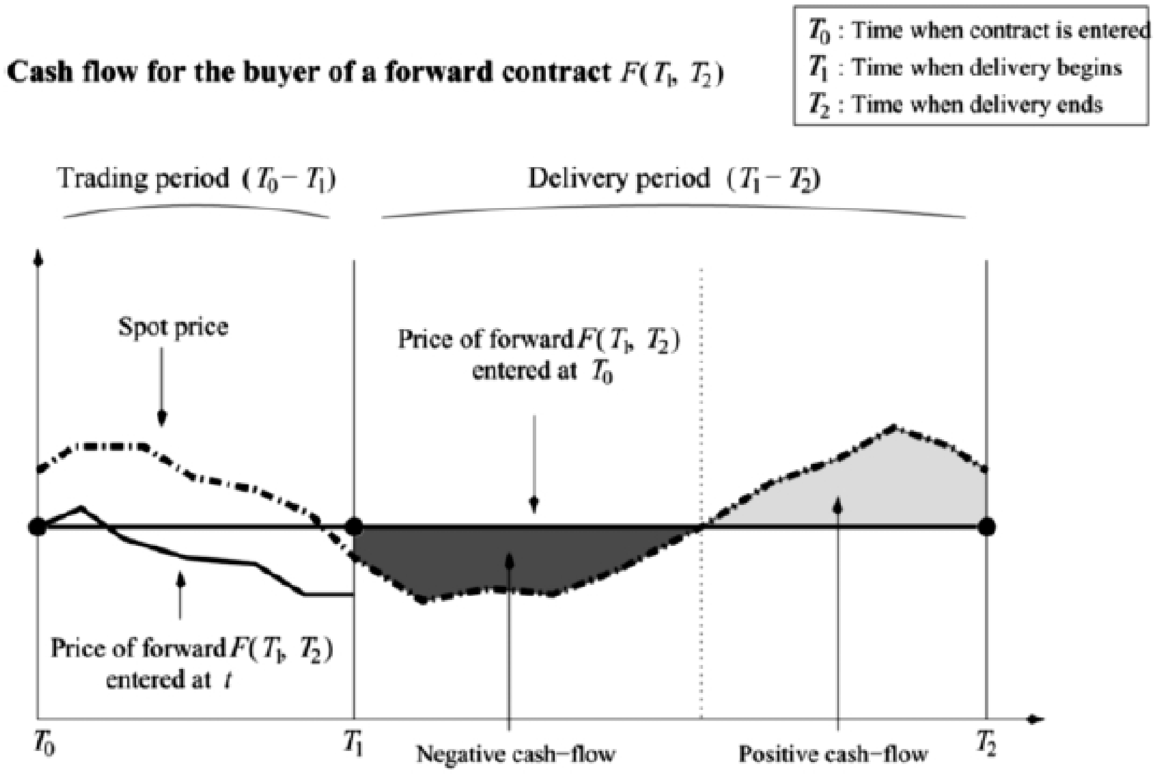
The total cash-flow for a hedged future position is shown in table 3. A short position will have a positive cash-flow, while a long position will have a negative cash-flow.

Total mark-to-market settlement
+
Total spot price reference
+
Total cash-flow by fulfilling obligation
=
Total cash-flow from hedging

Table 3: An overview of the how the cash-flow is calculated. The total cash-flow is the product of the previous calculations shown in equations 17, 19 and 21.

2.6 Hedging with forwards

In this framework, we keep the contracts until maturity. Because of this, forward and future contracts would have same total cash-flow if the maturity of the contracts is exactly the same (as long as we do not include interest rates). The difference between the future and forward is related to the timing of the cash-flow. The daily mark-to-market cash settlements do not occur but are instead accumulated and settled throughout the delivery period (Nord Pool 2010). Because of this, no settlement occurs before the delivery period. Graph 7 shows that the cash-flow during the trading period remains unchanged even though the forward price is volatile.



Graph 7: An overview over a cash-flow development of a forward contract at Nord Pool from acquisition and until expiration (Fleten 2003)

Equation 15 can also be used for illustrating the price of a forward at a specific time. As mark-to-market cash settlement does not occur, equation 16 and 17 are not applied for forwards because there are no daily mark-to-market settlements.

The accumulated mark-to-market is settled together with the spot price reference in the delivery period.

$$Mark\text{–}\ to\text{–}\ market\ cash\ settlement\ and\ spot\ price\ reference\ per\ day = \Big(\big(-(F\big(t_x^*, T_{1,}T_2\big) + (F(T_1))\big) - (\ -\big(F(T_1) -\ S_y\big))\Big) *\ h_d \tag{22}$$

This formula uses equations 16 and 18 combined together. This affects the cash-flow for each delivery day by closing out the accumulated mark-to-market differences and the regular spot price reference. The total mark-to-market cash settlement and spot price reference are shown in equation 23.

$$Total\ mark\text{–}\ to\text{–}\ market\ cash\ settlement\ and\ spot\ price\ reference = \sum_{Y=1}^{n} \Big(\Big(\big(-(F\big(t_x^*, T_{1,}T_2\big) + (F(T_1))\big) - (\ -\big(F(T_1) -\ S_y\big))\Big)\Big) h_d \tag{23}$$

To calculate the cash-flow for fulfilling a short or long obligation, we can use equations 20 and 21 as we did with the futures. The reason for this is that we sell or buy electricity on the spot market, regardless of the derivative contract used.

The total cash-flow for the forward hedge is shown in table 4.

Total mark-to-market and spot price reference
+
Total cash-flow from fulfilment of obligation
=
Total cash-flow from forward hedging

Table 4: An overview of the cash-flow for a forward hedge. The total cash-flow from a contract is the product of the previous calculations from equations 21 and 23.

2.7 Comparing a hedged position with an unhedged position

A way to measure if a particular hedge gives better results is to compare it with the alternative of remaining unhedged. Staying unhedged means that one does not use the future markets to hedge a position but instead purchases or sells power at the current spot price. To make the comparison, the time frame would need to exactly match the one we used under the hedged position. The unhedged position must therefore replicate the hedged position so that it purchases and sells the same total hours during the delivery period. From graphs 6 and 7, this consists of all delivery days between $T_1$and T_2. The formulas for calculating the cash-flow are exactly the same as those used in formulas 20 and 21.

$$Cash - flow\ per\ day\ unhedged\ position = S_y * \ h_d \qquad (24)$$

$$Total\ cash - flow\ for\ a\ unhedged\ position = \sum_{y=1}^{n} S_y h_d \qquad (25)$$

The main point of this framework is to determine how to calculate whether it is best to hedge a position or to remain unhedged. For a hedge to be better, the cash-flow for a hedge with a short position would need to be greater than the cash-flow of an unhedged position. Similarly, the cash-flow for a long position would need to be less than the cash-flow from the hedged position.

2.8 Correlation between future/forward contract prices and spot prices

The price of a contract can be regarded as the prediction of what the average spot price will be in a corresponding delivery period. A method to gauge the predictability of the price is the study of the correlation between them. The more correlation there is between the contract price and the average spot price, the greater indication that there is a stronger degree of correlation between them. It is not possible to draw any conclusive evidence by solely studying the correlation, but it gives us an indication of whether the future and spot prices move in the same direction for a given time period. This is important from a hedge point of view because less correlation indicates that the contract price does not move in the same direction as future spot prices for different contracts. This also indicates whether a hedge may prove to be very profitable or whether it is likely to cause losses, outcomes dependent on how the spot price moves for a predetermined future price.

To establish a correlation coefficient, we need to define a method for determining the average spot price for a specific future contract, in other words, the average spot price during a delivery period for a future contract. To do this, we use the theory of arithmetic mean:

$$\bar{S} = \frac{1}{n}\sum_{y=1}^{n} S_y \qquad (26)$$

We use portfolio theory and the Pearson correlation coefficient between future prices at different times in a trading period and the average spot price in the delivery period.

$$cov[F_x, \bar{S}] = E[(F_x - E[F])(\bar{S} - E[\bar{S}])] \qquad (27)$$

F_x, The contract price on a random day during the trading period. It is required that the same arbitrary day is used for all the different contracts. The average spot price is the corresponding average spot price in the delivery period for the same contract.

$$corr(F_x, \bar{S}) = \frac{cov(F_x, \bar{S})}{\sigma_{F_x}\sigma_{\bar{S}}} \qquad (28)$$

The table below shows how the different answers may be interpreted. Because contracts have longer trading periods, it is possible that correlation will be quite different depending on at what time a position is taken in a contract. Our hypothesis is that the further away from the delivery period we are, the more uncertain the market is as to what the spot price will be in the delivery period, an uncertainty which will be reflected in a lower correlation.

-1 Spot price moves in the opposite direction to the future price	0 Spot price and future price move in random unpredictable directions	1 Spot price moves in the same direction as the future price

A correlation of 1 implies that the contract price predicts the spot price fairly correctly. If that is the case, then entering a hedge is not necessary because the cash-flow is the same regardless. The closer the correlation is to zero, the less ability the contract price has for predicting what the spot price will be. This implies that the spot price may either move in a favourable or unfavourable direction given a short or long position. The closer the correlation coefficient moves to zero, the less information can be read out of the correlation coefficient with regards to future price predictability.

3. Analysis of hedging with an optimal hedge ratio

3.1 Finding a optimal hedge ratio

In the analysis, we use historical spot and weekly future prices from Nord Pool from the period between 01.01.2006 – 31.12.2009. The weekly future contracts at Nord Pool contain 168 MWh, but in this scenario we assume that one contract contains 1 MWh.

In this case, a power supplier is going to deliver electricity at time (t+1). That means that we do not have a delivery period because we sell all of the power right away in the spot market. Such a strategy can for instance be used by an electricity supplier that regularly delivers electricity to the residential market but has a surplus capacity that it sells in the spot market once a week. The supplier wants to hedge the position to minimise the risk. We assume that the power supplier is risk adverse and therefore wants to reduce the variance of the sale. In this analysis, we study the possibility of reducing the variance of the sale when hedging with an optimal hedge ratio compared with a regular hedge ratio. It is also important to note that we do not study the returns of the sale in our analysis of this scenario.

As explained in part 2.1, b refers to the hedge ratio and b* to the optimal hedge ratio. For every MWh we plan to sell, b indicates the amount of MWh we will sell in the futures market. If b>0, we sell future contracts at time t and hold a short position. If b<0, we buy future contracts, and we therefore take a long position. When b=0, we do not use future contracts, and when b=1, we sell the same amount of electricity in the future market as we plan to sell in the spot market. As an example, we can show how the future positions change with different hedge ratios. We assume that we are selling 100 MWh in the spot market.

	Spot market	**Future market**
b=0.5	100 MWh	50 MWh
b=1	100 MWh	100 MWh
b=1.5	100 MWh	150 MWh

Table 5: The effect on the future position when the hedge ratio changes, given that the spot position remains at 100 MWh.

First, we will show an example of how we calculate b*. We use five contracts from the first five weeks of 2006 during which we sell the future contract two weeks before the delivery. To close out the position, we purchase the same identical contract on the closing day of trade before selling the power at the spot market on the same day.

Contract	Bought	Sold	Spot t+1	Future t	Future t+1	Spot Change	Future Change	Change in basis
ENOW 03-06	**02.01 2006**	**06.01 2006**	36.29	41.40	36.88	-1.22	-4.52	-3.30
ENOW 04-06	**09.01 2006**	**13.01 2006**	54.25	39.88	48.13	8.77	8.25	-0.52
ENOW 05-06	**16.01 2006**	**20.01 2006**	40.85	40.23	41.70	2.96	1.47	-1.49
ENOW 06-06	**23.01 2006**	**27.01 2006**	43.38	46.25	44.72	-5.33	-1.53	3.80
ENOW 07-06	**30.01 2006**	**03.02 2006**	43.47	44.00	41.54	3.08	-2.46	-5-54
		Variance	43.64	7.35	17.43	27.85	24.68	12.13
		Covariance				16.16		

Table 6: The time at which the contracts are bought and sold, and the spot and future price at that time. We also see the change in the spot and future prices from the day of purchase and the day of sale. The change in basis represents the change in future price and change in spot price. The covariance is the relation between the change in spot price and the change in the future price, from t to t+1. The covariance is calculated using EXEL.

We now have all the information we need to calculate the b*. Using equation 9, we find that the optimal hedge ratio in this example is 0.65. This means that if we are going to sell 100 MWh in the spot market, we sell 65 MWh in the future market to minimise the variance of the sale. The next step is to calculate the variance of the hedge. To do this, we use equation 14.

Contract	**Bought**	**Sold**	**Spot t+1**	**Future t**	**Future t+1**	**b***	**Value of the hedge b***	**Value of the hedge b=1**
ENOW 03-06	**02.01 2006**	**06.01 2006**	36.29	41.40	36.88	0.65	33.35	31.77
ENOW 04-06	**09.01 2006**	**13.01 2006**	54.25	39.88	48.13	0.65	59.61	62.50
ENOW 05-06	**16.01 2006**	**20.01 2006**	40.85	40.23	41.70	0.65	41.80	42.32
ENOW 06-06	**23.01 2006**	**27.01 2006**	43.38	46.25	44.72	0.65	42.38	41.85
ENOW 07-06	**30.01 2006**	**03.02 2006**	43.47	44.00	41.54	0.65	41.87	41.01
		Variance	43.64	7.35	17.43		92.22	127.03

Table 7: The day we buy and sell the contracts. It also gives us the optimal hedge ratio and the value of the hedge we have calculated with the different hedge ratio.

We see that using the optimal hedge ratio in our example, we manage to minimise the variance of value of the hedge.

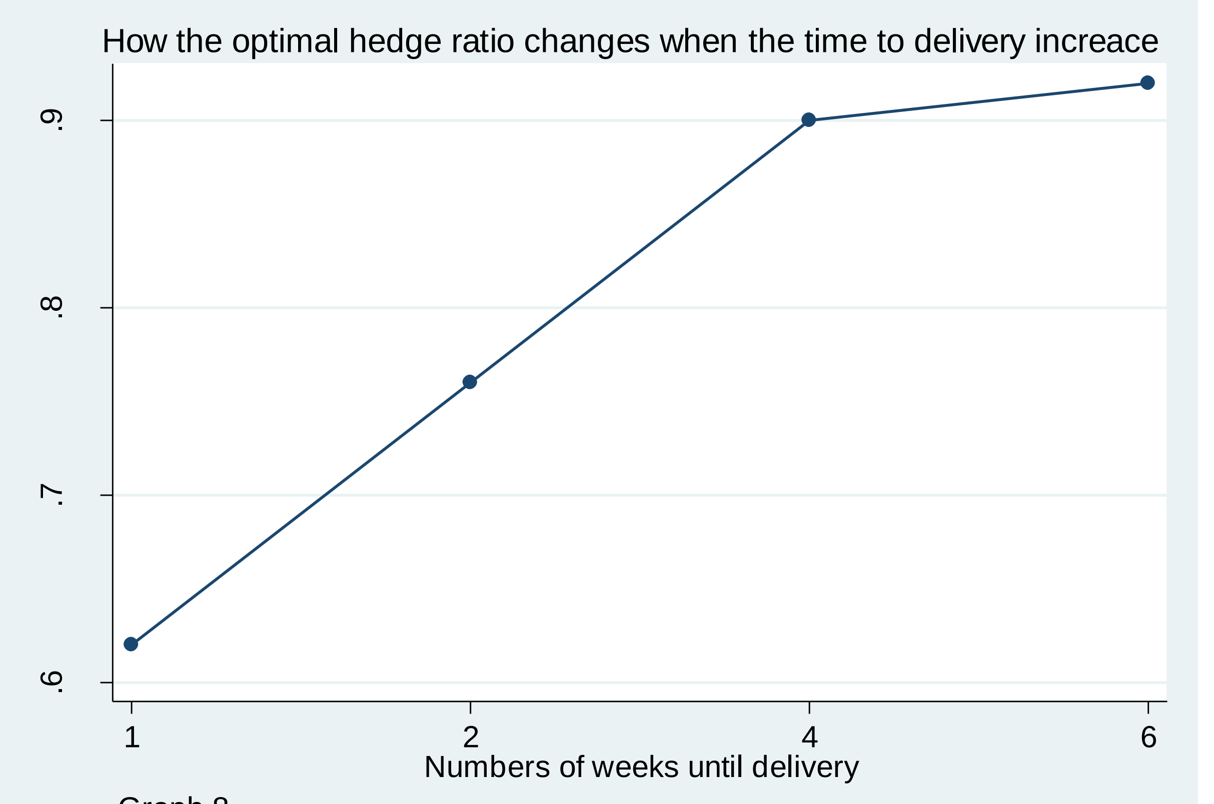
This was just an example of how we calculate the variance and optimal hedge ratio. The weekly futures have a trading period of six weeks. We have sold futures at different times during the trading period to see if the optimal hedge ratio changes if we enter a contract at different times. The times we have chosen are one, two, four and six weeks before we sell the power.

We have calculated optimal hedge ratios and the variance of the sale for all of the weekly futures contracts over a four-year period from 2006 to 2009, as we did in the example.

Total	Variance (b)	Optimal b (B*)	Variance (b*)
1 week	170.44	0.62	163.26
2 week	205.75	0.76	189.98
4 week	261.27	0.90	249.51
6 week	343.50	0.92	324.02

Table 8: The difference in the variance of the sale using a regular hedge ratio compared with the optimal hedge ratio b*. The number of weeks indicates the time until the sale.

If the supplier at time t wants to sell 100 MWh at (t+1) in the Nord Pool spot market and wishes to minimize the variance of their sale, they should hedge by selling 62 MWh in the futures market one week prior to the delivery. Likewise, it can hedge by selling 76 MWh in the futures market two weeks prior to the delivery. We see that the greater amount of time before the delivery, the larger amount we sell in the futures market to minimise the variance.

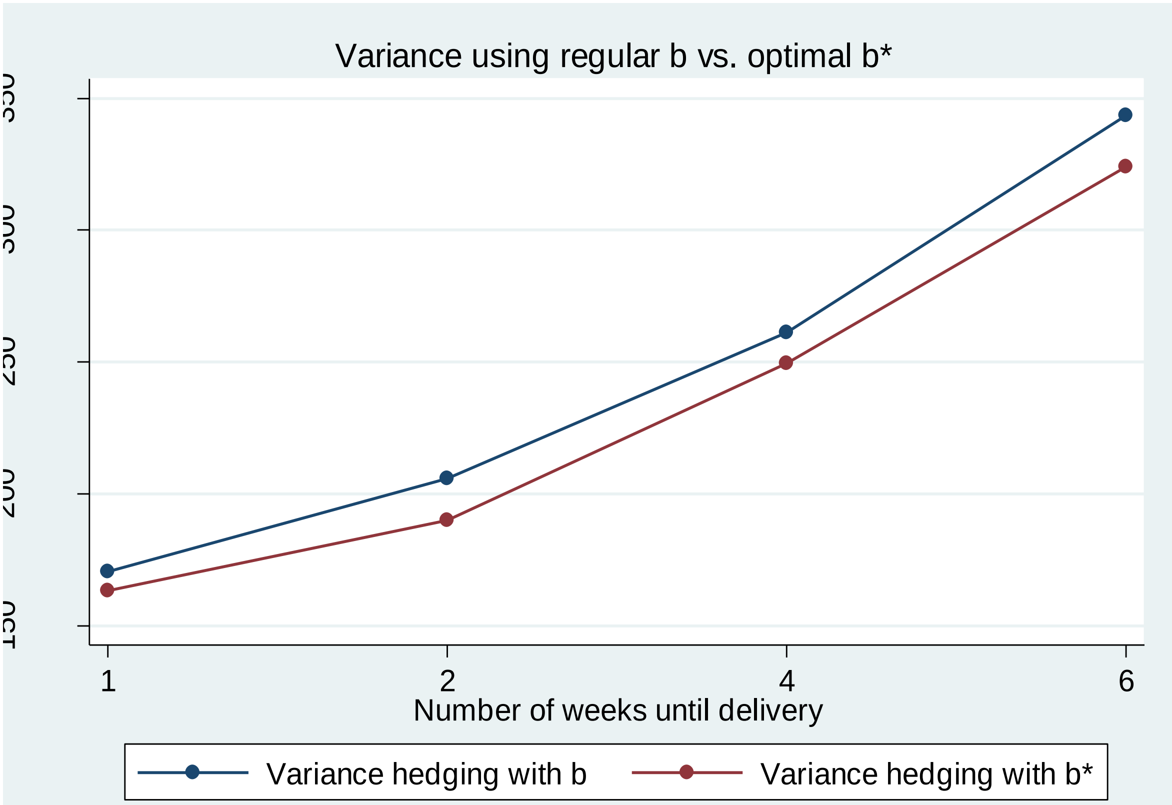


Graph 8

Graph 8 shows that as time prior to the delivery increases, the optimal hedge goes towards one. Using six-weeks-until-delivery futures, we see that when the supplier wants to minimise the variance of the sale, the optimal ratio is very close to one.

A high optimal hedge ratio close to one indicates that either the covariance between the spot and future price is high, or the variance of the future price is low. The consequence of this is that we sell nearly the same amount in the futures market for every MWh we plan to sell in the spot market. We see this especially in the four- and six-week-until-delivery contracts, where the optimal hedge ratio is over 0.90. If the optimal hedge ratio tends to be low, like we see in the one- and two-week-until-delivery futures, either the variance of the future price is high or the covariance is low. A high variance in the future price tends to lower the position that we take in the futures market because a large future position makes the total position more risky.

The optimal hedge ratio changes when the difference between the covariance of spot and future prices and the variance of the future prices change. If the percentage gap between the two increases, the optimal hedge ratio approaches zero. As the percentage gap gets smaller, the optimal hedge ratio approaches one.



Graph 9

The main goal of calculating an optimal hedge ratio is to compare the performance of a fully hedged position to an optimally hedged position. From table 8, we see that the variance of the optimal hedged position is lower than the fully hedged position. Our risk-adverse power supplier using the optimal hedge ratio will reduce the variance of the value of the hedge and thus the final sale of power. Graph 9 shows how much variance the power supplier can reduce with the optimal hedge ratio.

What we also see in graph 9 is that the variance increases significantly as the optimal hedge ratio approaches one. This also shows that the greater the length of time remains to the delivery period of the electricity future, the greater the variance. A higher variance of the value of the hedge indicates that if you hedge over a longer period, the electricity price can become more volatile than it is in the short run. When the variance increases, it becomes more and more difficult for the power supplier to determine the revenues.

3.2 The effectiveness of using the optimal hedge ratio

In the section 3.1, we found that there is an optimal hedge ratio that can minimise the variance of the value of the hedge. In this section we study how much risk you can eliminate from your position by hedging with b* rather than the regular b.

Total	**Variance (b)**	**Variance (b*)**
1 week	170.44	163.26
2 week	205.75	189.98
4 week	261.27	249.51
6 week	343.50	324.02

Table 9: The variance of the value of the hedge using the optimal ratio b* and the regular ratio b.

These results are the same that we presented in table 8. We use the numbers to determine how much risk the power supplier can eliminate by hedging optimally. To calculate this, we can use either equation 11 or 13 because both give us the same answer. Because we have the variance of both ratios, we focus here on equation 11. The results are presented in table 10.

	Effectiveness (e)
1 week	-0.044 = -4.4%
2 week	-0.083 = -8.3%
4 week	-0.047 = -4.7%
6 week	-0.060 = -6.0%
Average:	-0.058 = -5.8%

Table 10: The hedging effectiveness calculated using the optimal hedge ratio b* instead of the regular hedge ratio b. The table also indicates the average (e) of the different times until delivery.

All the numbers are negative, which means that an investor can reduce risk by hedging with an optimal hedge ratio. Had the effectiveness been positive, investors would benefit from taking more risk than with the optimal hedge ratio.

Table 10 shows that we can reduce risk with the optimal hedge ratio and that we can reduce the most risk when using two-week-until-delivery futures. If we take the average of all the futures, we get an effectiveness of -5.8%. This means that a power supplier could reduce the risk of achieving spread out values of a sale of power by hedging with an optimal hedge ratio over the four-year period that we studied.

3.3 Summary

Our scenario was a risk-adverse power supplier that was going to deliver electricity at time (t+1) and wanted to hedge the position in the futures market to reduce risk. We wanted to find an optimal hedge ratio that reduced the variance of the hedge. The results that we show follow the model from sections 2.1–2.3 to minimise the variance of our position. By calculating an optimal hedge ratio, we manage to reduce the risk of the end value of a hedge using future contracts at Nord Pool.

Our results show that, for the futures market, the optimal hedge ratio lies below one, meaning that investors should sell a larger position in the spot market than they sell in the futures market. When we use shorter until-delivery futures, the optimal hedge ratio goes towards zero. The longer until-delivery futures we use, the closer the optimal hedge ratio goes towards the regular hedge ratio one.

We also studied the effectiveness of using an optimal hedge ratio compared to a regular hedge ratio. We determined that, on a four-year perspective, the average risk that we can reduce when hedging with the optimal hedge ratio is 5.8%.

4. Hedging at Nord Pool with delivery analysis

The previous hedge strategy showed that it is possible to minimise the risk of a position by minimising the variance through an optimal hedge ratio. That strategy did not study the hedge returns at all. In this analysis, we use the previously explained hedge strategy from part 2.4 and compare the returns for a fully hedged position to an unhedged position when holding a contract until maturity.

Our primary focus will be the short position of a hedge where we hedge a sale of 1 MWh per hour of each day during one year for all of the years from 2005 to 2009 by selling a contract when it first opens for trade and by holding the contract throughout the delivery period. We will explain the results we determine from this thoroughly. However, we will also show how the cash-flows evolve if we change the entry time for a contract. The times chosen are halfway through the trading period and at the closing day of the trading period.

The sale of power is performed through Nord Pool at the current spot price for each day. The spot price is the same as the system price which was described in chapter 1.6. The reason for the specific time limitation is that, from 2005, the price of power became specified in Euros instead of Norwegian Kroners, and we wished to avoid problems with differing currencies. The different types of contracts that we have studied are weekly futures, monthly forwards, quarterly forwards and yearly forwards.

The objective of this study is, as previously explained, to locate any clear trends as to whether hedging yields better results than not hedging when measured in cash-flow for the specific hedge strategy. It is also our intention to see if there are any repetitive trends for specific contracts for the different years.

4.1 Weekly Futures

We will first show how the cash-flows are calculated, with the theory we showed in Chapter 2.2, for a real contract that had been listed at Nord Pool. The example contract is ENOW01-06. The future contracts are available for trade six weeks before delivery, so for this specific contract the trading period was that between 21.11.2005 – 30.12.2005, and its corresponding delivery period was from 02.01.2006 – 08.01.2006. We simulate taking a short position in the future on its first day of trade; with equation 15, we show that the future price is 39.98 Euros.

The daily margins are calculated for the first few days with equation 16 and are shown in table 11. Due to the number of trading days, we have taken a sample of the first five days only. The total mark-to-market settlement for ENOW01-06 is calculated using equation 16 and is shown in the same table.

Date	In days	t_x	Future price	Daily margins	Total margin
21.11.2005	0	t_0^*	39.98		
22.11.2005	1	t_1	40.00	-3.36	
23.11.2005	2	t_2	39.83	28.56	
24.11.2005	3	t_3	39.03	134.4	
25.11.2005	4	t_4	38.80	38.64	
28.11.2005	5	t_5	40.40	-268.8	
30.12.2005	28	t_{28} **OR** $F(T_1)$	36.5		
Total mark-to-market settlement					584.64

Table 11: An overview of the mark-to-market settlement of ENOW01-06. Since trading in the market only occurs on weekdays, we have chosen to specify the number of days in the contract instead of using dates.

4.2 Spot price reference

In the delivery period, there is a spot price reference that is calculated daily by equation 18. This is shown in table 15. We use equation 19 to calculate the total spot reference.

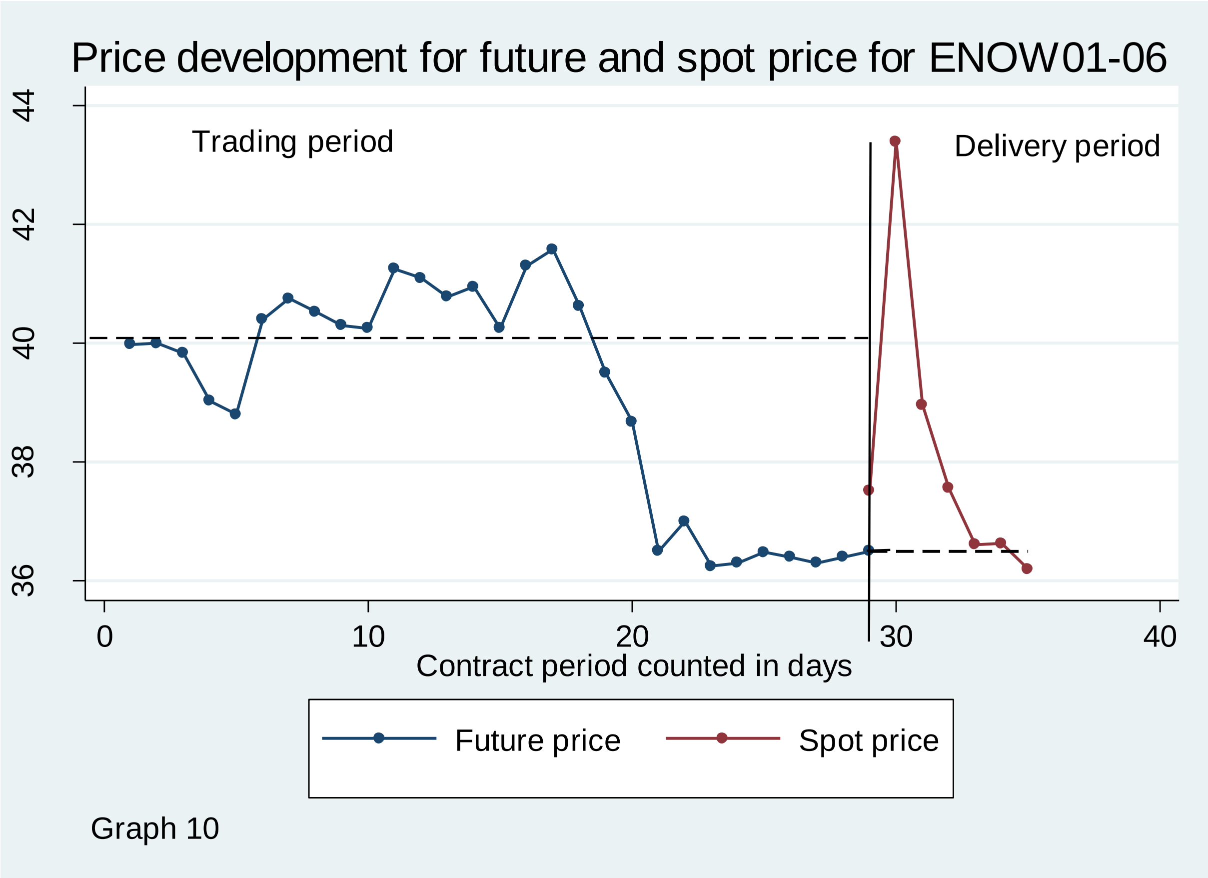
Date	In days	t_x	S_y	Future contract closing price	Spot price	Spot price ref
02.01.2006	29	t_{29}	S_1	36.50	37.51	-24.26
03.01.2006	30	t_{30}	S_2	36.50	43.39	-165.37
04.01.2006	31	t_{31}	S_3	36.50	38.96	-59.14
05.01.2006	32	t_{32}	S_4	36.50	37.56	-25.44
06.01.2006	33	t_{33}	S_5	36.50	36.61	-2.73
07.01.2006	34	t_{34}	S_6	36.50	36.63	-3.04
08.01.2006	35	t_{35} **OR** $F(T_2)$	S_7	36.50	36.20	7.14
	Total spot price reference					-272.84

Table 15: An overview of how the spot price reference affects the cash-flow of ENOW01-06 during the delivery period.

4.3 Graphical illustration

Graph 11 is a graphical illustration of how the prices change during the trading period and the delivery period. For a short position, a negative slope for the future price yields a positive daily margin. A positive slope yields a negative margin. Because we keep the position throughout the whole contract, it is the closing value of the trading period that is of the most importance. The dashed black line is the future price at the purchase of the contract. If the closing future price has a lower value then the dashed black line, the total mark-to-market settlement is positive. For this specific contract, this is the case, as shown by our previous calculations.

Because we have already received the difference in payment between the opening and closing prices of the future contract, the spot price reference yields negative payment if the corresponding spot price is higher than the closing value. We have the opposite case if the closing value is lower than the spot price. The closing value is represented by a dashed line in the delivery period. In this specific contract the average spot price is well above the closing value, as shown by our calculations and illustrated through the graph.



A graphical illustration of how the future and spot prices for ENOW01-06 change during the trading and delivery period, respectively. The blue line represents the future price and the red line represents the spot price.

4.4 Calculation of cash-flow

In the delivery period, we use equations 20 and 21 to calculate the total cash-flow from the sale of the power. This is shown in table 16.

Date	In days	t_x	S_y	Spot price	Hours	Cash-flow
02.01.2006	29	t_{29}	S_1	37.51	24	900.26
03.01.2006	30	t_{30}	S_2	43.39	24	1041.37
04.01.2006	31	t_{31}	S_3	38.96	24	935.14
05.01.2006	32	t_{32}	S_4	37.56	24	901.44
06.01.2006	33	t_{33}	S_5	36.61	24	878.73
07.01.2006	34	t_{34}	S_6	36.63	24	879.04
08.01.2006	35	t_{35} **OR** $F(T_2)$	S_7	36.20	24	868.86
Total cash-flow from selling power						6404.84

Table 16: An overview of the cash-flow received from selling power using a hedged contract. Notice that we sell the power at the corresponding spot price in the market.

The total cash-flow for ENOW01-06 is summarised in table 16.

Total mark-to-market settlement	**584.64**
Total spot price reference	-272.84
Total cash-flow from delivery of power	6404.84
Total cash-flow from hedging	6716.64

Table 17: An overview of the previous calculations to show the total cash-flow from a hedged position.

The cash-flow of an unhedged position would, for this contract, be same as the total cash-flow from selling power in table 16. We see that, for this specific contract, hedging gave better results than not hedging as measured in cash-flow. An easy way to show the reason for this is that the future price when acquired is higher than the average spot price in the delivery period.

This is illustrated in graph 10, where the average spot price is higher than the future price for only one delivery day.

4.5 Empirical results

This calculation method has been used here to study all base load weekly future contracts at Nord Pool from 2005 to 2009 and is presented on a per year basis. Each year represents the total cash-flow of the 52 different weekly contracts that were available for trade, each corresponding to hedging and delivering 1 MWh per hour for a whole year. The results are shown in table 18.

2006			**2008**		
	Hedged	451592		Hedged	430740
	Unhedged	424569		Unhedged	391093
	Difference	27023		Difference	39647
	Percent	6 %		Percent	9 %
2007			**2009**		
	Hedged	271182		Hedged	321987
	Unhedged	242879		Unhedged	312583
	Difference	28303		Difference	9405
	Percent	10 %		Percent	3 %
Total					
	Total hedged	1475502			
	Total unhedged	1371124			
	Difference	104378			
	Percent	7 %			

Table 18: A summary of the cash-flow from a short position in a hedged contract delivering 1 MWh per hour for a whole year. The percentage difference is calculated using the cash-flow from the hedged position as the baseline number. The total of hedged and total unhedged represents the total cash-flow for all years.

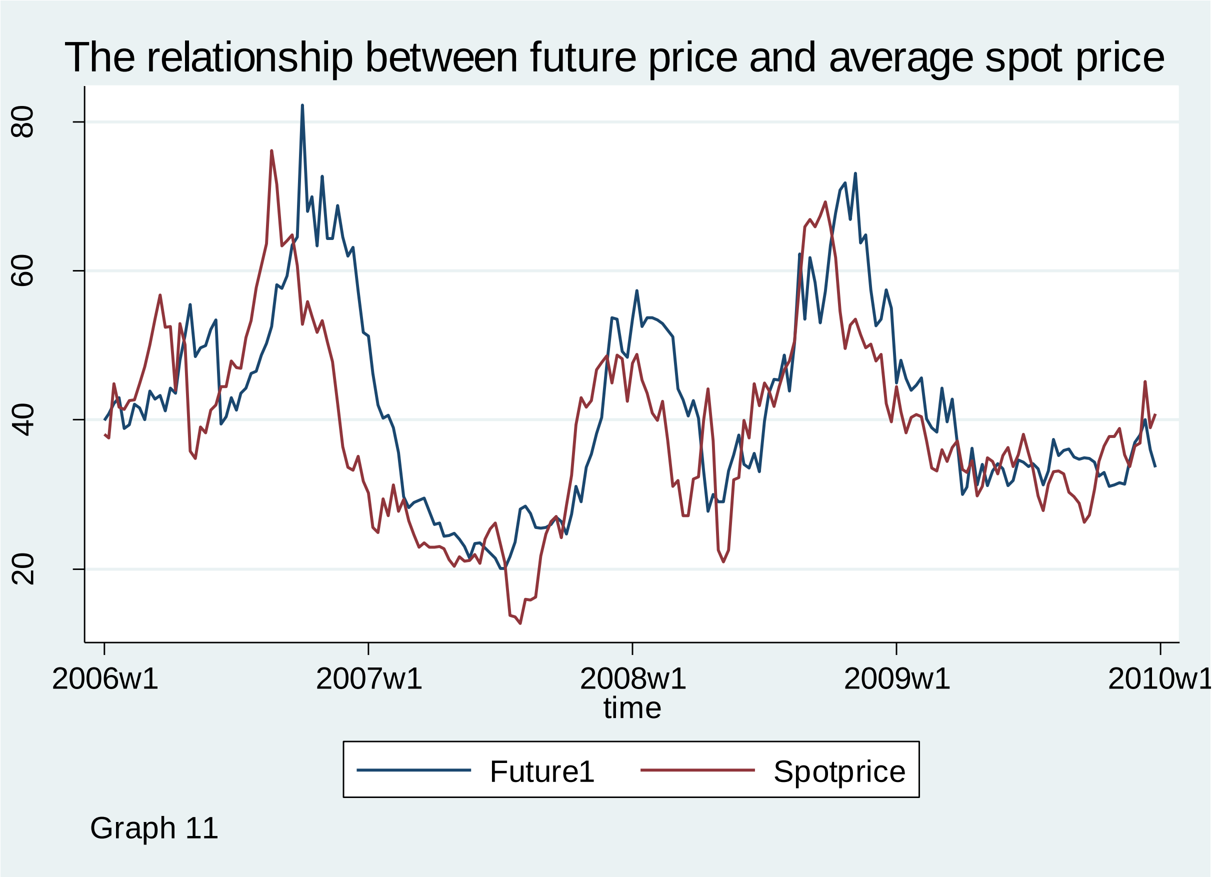
The results from this data indicate that the Hedging strategy maximises returns when compared with not hedging at all. This is supported in all the years from 2005 to 2009, with an average percentage difference of 7%. It is interesting to see whether there is a general explanation for this relatively large difference. In our previous theory, a positive difference meant that the product of the mark-to-market settlement and the spot price reference is positive. A decomposition of these is shown in table 19.

	Spot price ref	**Mark-to-market**	**Total**
2006	5647	21066	26712
2007	4223	23273	27496
2008	6390	34635	41025
2009	1282	7862	9145
	17542	86836	104378

Table 19: An overview of the total difference between a hedged and unhedged cash-flow, decomposed in spot price reference and mark-to-market settlement.

The majority of the difference is due to the mark-to-market daily settlement in the trading period of the contract. This indicates that there is an average decline in the future price from the opening of the contract and until the expiration, which for a short position results in a positive margin payment. The spot price reference is also positive, but constitutes very little of the total difference. This signals that, on average, the closing price of a contract predicts the average spot price during a delivery period fairly accurately. The fact that it is a not a negative number points to the fact that, on average, the spot price has not been higher than the closing price of a contract. This is important because it means that the opening price of a contract has, on average, been overpriced compared with the average spot price in a delivery period, illustrated by the positive mark-to-market settlement and a positive spot price reference.

To illustrate this graphically, we used equation 26, calculated the average spot price for each delivery period and combined it with the corresponding opening price of the future contract for that delivery period. Graph 11 shows that the spot price is, for the majority of the time period, lower than the opening price, with the exception of the first half of 2006 and other shorter time periods. There are signs from this graph that the relationship has become more stable for 2009, which might mean that the future price is more correct, and/or that the spot price has been less volatile.



The relationship of an opening price of the contract denoted as Future1 compared with the average spot price in the delivery period termed Spotprice.

4.6 Correlation

Since we have 206 contracts, this enables us to use the correlation coefficient explained in equation 27 to quantify the relationship between the opening price of the contract and the closing price against the average spot prices for all four years. The results are shown in table 20.

	Future 1	**Spot price**			**Future 2**	**Spot price**
Future1	1			Future2	1	
Spotprice	0.7326	1		Spotprice	0.9785	1

Table 20: Future 1 denotes the opening price of a contract, Future 2 denotes the closing price of a contract. Spot price is the average spot price of a contract.

The correlation between future 1 and the spot price shows that there is some degree of correlation between them. From that correlation, we can interpret that for the different opening values of the future contract, the spot price will to a certain degree have the same value in the delivery period. For a correlation to be perfect, the correlation coefficient must be 1. Our results show that there is some degree of uncertainty with this relationship. It is not possible to use these results to say that a hedged short position will be profitable because the correlation coefficient cannot tell us if the opening future price is higher or lower compared with the spot price, even though our data have shown that this is the case. The reason for this is that the closer the coefficient is to 0, the less correlation we have between two values, which means that the future price and spot price move in random directions.

For the relationship between future 2 and spot price, we have a correlation that is very close to perfect. This strong correlation can be interpreted as the product of the fact that the closing price of a contract is a fairly good predictor of the average spot price in the delivery period. As our data show, this is also the case.

4.7 Forward contracts

We used the same hedge strategy for all the base load forward contracts available at Nord Pool. Because we hold a contract from the opening and until the delivery, the total cash-flow is the same as with futures. The difference with forwards is related more to the timing of the cash-flow. More specifically, the daily mark-to-market difference is not settled before each day in the delivery period, as shown by equation 22. We will therefore not use a forward contract to present how the calculation is done because this can be understood from the example of the future contract. The results from the simulation are presented in table 21.

		1 Month	**Quarterly**	**Yearly**
2006				
	Hedged	364079	233664	210240
	Unhedged	425674	425674	425674
	Difference	-61595	-192010	-215434
	Percent	-17%	-82%	-102%
2007				

	Hedged	388816.6	234800.4	232140
	Unhedged	244743	244743	244743
	Difference	144074	-9942	-12603
	Percent	37%	-4%	-5%
2008				
	Hedged	427972.3	319576.6	238601.3
	Unhedged	392870	392870	392870
	Difference	35102	-73294	-154269
	Percent	8%	-23%	-65%
2009				
	Hedged	422262.7	367345.9	317812.8
	Unhedged	306750	306750	306750
	Difference	115513	60596	11063
	Percent	27%	16%	3%
	Total hedged	1603131	1155387	998794
	Total unhedged	1370036	1370036	1370036
	Difference	233094	-214650	-371242
	Percent	15%	-19%	-37%

Table 21: A presentation of the results from simulating a short position that sells 1 MWh per hour each year at the average spot price each day. This delivery is hedged with different types of forward contracts that correspond to the delivery time of power. The different forward contracts were sold when they were first available for trade. We use the formulas presented in the theory to calculate the total cash-flow for the hedged and unhedged positions. For the hedged position, the cash-flow is the product of equations 21 and 23. For the unhedged position, the cash-flow is calculated from equation 21. The results of "difference" and "percent" are calculated using the hedged position as the baseline number.

4.7.1 Monthly forwards

The monthly contracts are traded six months before the delivery period. Apart from 2006, when we had relatively high spot prices, this strategy yields better returns compared to not hedging for all other years. For all the years in total, the difference is €233,094, which is 15% better then the unhedged cash-flow. That is a considerable amount, a fact that shows that the market has consistently overpriced the opening forward price for monthly forwards on average.

	Spot price ref	Mark-to-market	Total
2006	4217	-65811	-61595
2007	18988	125086	144074
2008	19751	15352	35102
2009	15190	100323	115513
Sum	58145	174949	233094

Table 22: An overview of the total difference between a hedged and unhedged cash-flow, decomposed in spot price reference and mark-to-market settlement.

Table 22 shows how the mark-to-market settlement for the forward price has, apart from 2006, had a negative slope. Since the spot price reference is positive for all years, the average closing price of the monthly forward has, on average, been higher than the average monthly spot price in the delivery period. Therefore, these results indicate that, on average, the opening prices of forward monthly contracts have been overpriced and that the hedge strategy that we have presented has historically given greater returns.

4.7.2 Quarter forwards

The quarterly forwards are traded two years prior to the delivery period. As table 21 shows, hedging an obligation two years ahead with quarterly forwards when it opens for trade has given negative returns when compared with an unhedged investment. That means that the opening prices have been considerably underpriced.

	Spot price ref	Mark-to-market	Total
2006	28093	-220102	-192010
2007	32740	-42683	-9942
2008	48702	-121995	-73294
2009	14759	45837	60596
Sum	124294	-338944	-214650

Table 23: An overview of the total difference between a hedged and unhedged cash-flow, decomposed in spot price reference and mark-to-market settlement.

From table 23, it is clear that, on average, there has been an increase in the forward price for all forward contracts in the trading period for all years apart from 2009 that yields negative margins in the delivery period. Because the spot price reference is positive, the average quarterly spot price has been lower than the closing price of the contracts. For the years 2006–2008, it is clear that hedging an obligation with quarterly forwards two years before delivery is a worse option then remaining unhedged. However, it is, however, interesting that for the year 2009, it is the better option, which might mean that in the future the market will have a higher opening price for quarterly contracts.

4.7.3 Yearly forwards

The yearly forwards are traded three years prior to delivery period. As table 20 shows, the results from hedging through yearly forwards give less cash-flow then remaining unhedged. The patterns for yearly forwards are very similar to quarterly forwards but the differences are even greater.

	Spot price ref	Mark-to-market	Total
2006	-98400	-117034	-215434
2007	78501	-91104	-12603
2008	46072	-200341	-154269
2009	32087	-21024	11063
Sum	58260	-429503	-371242

Table 24: An overview of the total difference between a hedged and unhedged cash-flow, decomposed in spot price reference and mark-to-market settlement.

The mark-to-market is negative for all years, which shows that there has been a positive slope for the forward price during the trading period presented in table 24. Apart from 2006, there is a positive spot price reference. This means that the closing price has been underpriced compared with the average yearly spot price for all years apart from 2006. As we saw with quarterly forwards, the yearly forwards for 2009 yield better cash-flow for the hedged position compared with the unhedged position and may as stated represent a new trend in the future.

4.8 Hedging the position at different times:

From the previous calculation, it is clear that shorter maturity contracts are preferred before longer maturity contracts if you were to hedge an obligation as soon as a contract is available. As an extension of this, we wished to see if this trend remained the same if we changed the time of trading in the trading period.

The calculations that have been done are exactly the same as previously, but we are limiting the study to show only the returns versus the unhedged cash-flow. The unhedged cash-flow is, of course, exactly the same as previously, the only difference for the hedged cash-flow is that the returns change due to different mark-to-market settlement because we engage at a different time than the opening day of trade.

Table 25 shows how the returns change if we engage in a contract halfway through the maturity of a forward or future. For monthly forwards it is 3 months before delivery, quarterly it is one year before delivery, yearly it is 1.5 years before delivery and futures it is 3 weeks before delivery.

	Monthly Forward	**Quarterly Forward**	**Yearly Forward**	**Weekly Future**
2006	-3.09%	-84.55%	-62.52%	5.06%
2007	25.82%	28.37%	18.76%	4.72%
2008	11.87%	-10.95%	-2.19%	6.82%
2009	13.24%	32.57%	24.37%	1.31%
Total	11.16%	0.82%	-1.24%	4.69%

Table 25: An overview of the returns from hedging compared with not hedging. This table is a simplification of table 21 in which we only show the "returns" when selling the contract halfway through the trading period.

The trend we see is that the returns from the monthly forward and weekly futures are reduced compared with our previous calculations, while the quarterly and yearly forwards now have nearly the same return as an unhedged position.

The other trading time we have calculated is by selling a contract on the last day of the trading period. The result from this is shown in table 26.

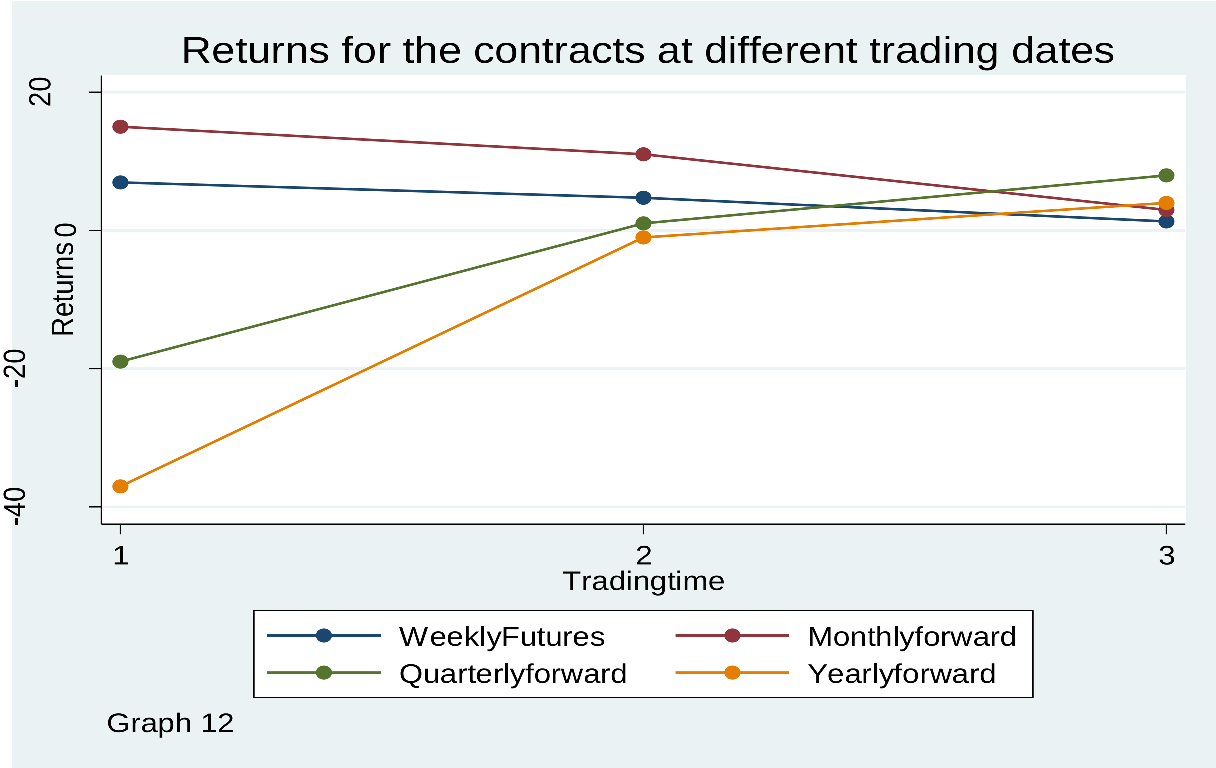
	Monthly Forward	Quarterly Forward	Yearly Forward	Weekly Future
2006	1.60%	6.19%	-30.07%	1.38%
2007	5.63%	11.80%	24.29%	2.03%
2008	3.09%	11.03%	10.50%	1.52%
2009	1.97%	4.59%	9.47%	0.49%
Total	2.85%	8.32%	4.08%	1.34%

Table 26: An overview of the returns from hedging compared to not hedging. This table is a simplification of table 21 in which we here only show the "returns" when selling the contract at the closing day of the trading period.

These results continue the trend that the previous table showed. The shorter maturities, such as weekly futures and monthly forwards, become very close to being the same as remaining unhedged. These results are in line with what we saw when investigating the correlation of the closing contract price and the average spot price in the delivery period for weekly futures. This underlines that, for shorter maturity contracts, the closing prices are a fairly accurate predictor of the average spot price. For the longer maturity contracts, the average returns of hedging are now a better option than staying unhedged. This shows that the closing prices of these contracts are overpriced when compared to the average spot price at the closing day in the trading period.

Graph 12 summarises all the returns for the cash-flow compared with an unhedged cash-flow at different trading times. The trends we see are fairly interesting where the closer to maturity we are, the better the longer maturity contracts are. It may well be that these longer maturity contract have not been traded a lot in the early parts of the trading period. A study of the open interest of these contracts would reveal how much contracts have been traded at all and thus reveal if these types of contracts have been as liquid, as for instance, the weekly and monthly contracts. Another reason might be that the average spot prices before our timeframe have been generally lower than what we have had in 2005–2009, which explains why we have had low opening prices. As previously mentioned, the results from 2009 could be an indication that the opening price of longer maturity contracts will be more precise in the future.

The shorter maturity contracts returns generally decline the closer we are to the expiration of the contract. This justifies the hypothesis that on average, the closing values of shorter maturity contracts predict the average spot price fairly accurately, but has historically been overpriced by a few percent compared with the average spot price.



A summary of the return of cash-flows for different contract compared to the cash-flow of not hedging at all with different trading times. The graph uses the results from returns that are calculated in tables 18, 21, 25 and 26.
Trading time 1 = Selling a contract the first day of trade.
Trading time 2 = Selling a contract halfway through the trading period of a specific contract.
Trading time 3 = Selling a contract on the final day of the trading period for the specific contract.

4.9 Summary

Our results provide a few interesting interpretations. We have calculated the cash-flow from selling electricity for each hour for a whole year and hedged delivery with different types of contracts. These contracts have been sold as soon as they were available in the market for the given price at that time and have been thoroughly discussed. We have also briefly shown how the cash-flow changes when selling the same contract as previously but at a later trading time.

If a power supplier was to base a hedge strategy for yearly delivery of power based on this ex-post analysis, then we would advise them to sell a shorter maturity contract the instant they are available for trade. Shorter maturity contracts are preferred for most of the trading period, but the closer we are to the closing day of the contracts, the better the longer maturity contracts are. As shown in graph 20, the quarterly and yearly contracts give better returns if sold on the closing day of the trading period if compared with the closing day of shorter maturity contracts.

There is also some interesting information obtained from the spot price reference that has been calculated for all the contracts. Apart from the yearly forward 2006, the spot price reference values are always positive. This result can be interpreted to indicate that a hedge strategy in which you hedge your delivery commitments at the closing day of trade will also be a better option than staying unhedged. The difference in return is particularly large for monthly and quarterly forwards.

5. Conclusion

In this thesis we have studied two specific hedge strategies at Nord Pool that can potentially be used as a risk management tool. The first strategy was a study if it is possible to reduce the variance of a sale of power through a risk-minimization strategy with future contracts. The other strategy studied how the returns of sale of power were affected by using different future and forward contracts and comparing this to the alternative of not hedging the sale at all.

Our thesis has shown that an optimal hedge ratio strategy can be used to minimise variance and therefore minimise the risk for a firm. The results show that the optimal hedge ratio in most cases is below 1, which means that we take a larger position in the spot market compared with the future market. By hedging with an optimal ratio, the risk of a position is lower than if we were unhedged. This was illustrated by the hedge effectiveness.

The specific hedge strategy we studied proves that, historically, it has been better to hedge an obligation with shorter maturity contracts. Weekly futures and monthly forwards yield greater returns then the alternative of staying unhedged. Longer maturity forwards are proven to give a negative cash-flow compared with staying unhedged unless one hedges the position on the final days of the trading period for these contracts.

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> This paper describes the approach to and experience of the deregulation and liberalisation of the Norwegian electricity sector from 1991. The Norwegian electricity market was subsequently integrated with the Swedish, Finnish and Danish markets to become the Nordic electricity market: the first common, integrated, intercountry electric power market in the world. The article discusses the background to electricity market reform, the analytical and legal foundations for reform and the chosen market and regulatory design. It finds that the market has performed well in terms of economic efficiency and market functionality, even when exposed to severe supply shocks because of water shortages for a power system that relies heavily on hydropower. However, it also identifies issues and challenges that must be addressed to improve the performance of the Nordic electricity market and its regulatory system.

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